

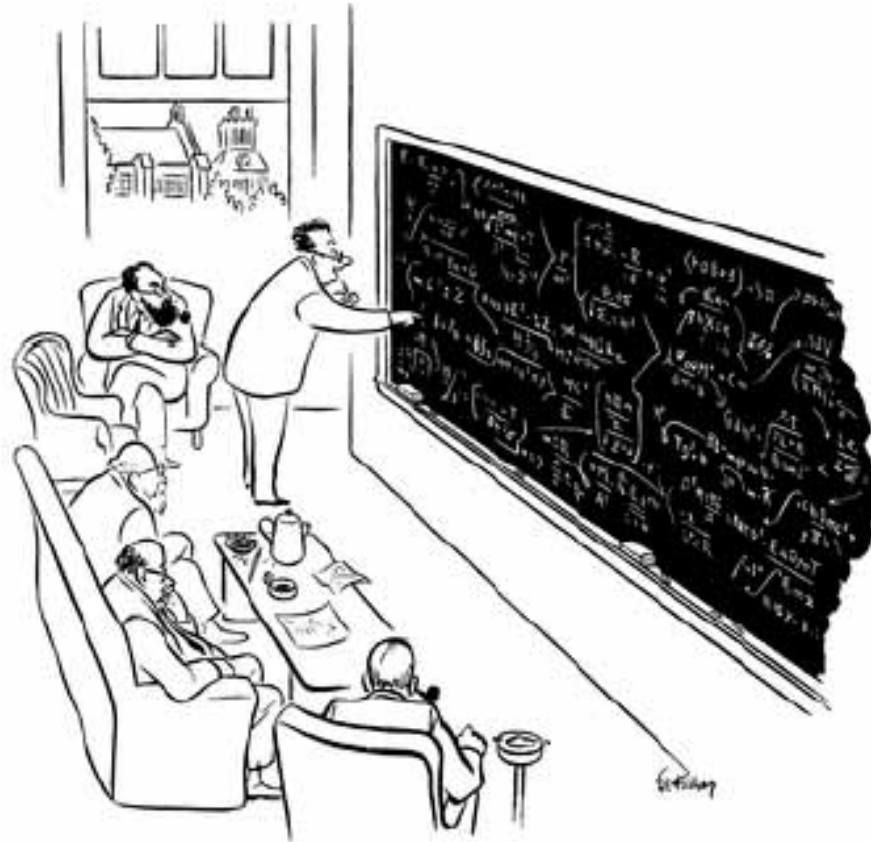
ELECTROMAGNETIC MODELLING OF MICRO-NANO STRUCTURED SURFACES

Jean-Philippe Colombier

jean.philippe.colombier@univ-st-etienne.fr

How to simulate light action ?

- Models
- Approximations
- Numerical methods

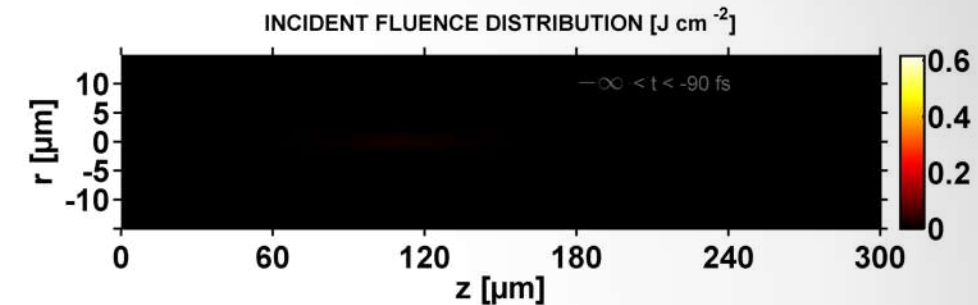


"Say, I think I see where we went off. Isn't eight times seven fifty-six?"

Different numerical approaches

➤ Beam Propagation Method

Long distance propagation for short λ (fibers ; transport...)
Paraxial approximation of the Helmholtz equation
Pseudo-spectral approach



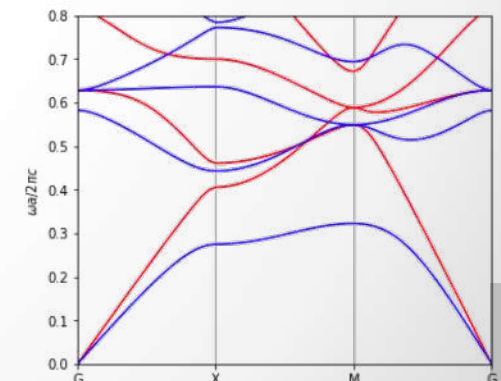
➤ Finite Difference Time Domain

Short distance propagation for short λ (Nanophotonics, Interference, Interaction...)
Long distance propagation for large λ (Radar, Microwave...)
Full Maxwell equations in time-domain approach



➤ Plane Wave Expansion

No propagation (Modes, stationary optics...)
Band structure based on Fourier modes (Photonics crystals)
Full spectral approach



I The paraxial equation solved by BPM method

Starting from the wave equation for a monochromatic wave

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} + k^2(x, y, z)E = 0$$

With the spatially dependent wavenumber

$$k(x, y, z) = k_0 \bar{n}(x, y, z)$$

The main assumption is that the variation due to propagation along the z axis represents the fastest variation in the field E . This variation is exemplified by introducing slowly variation in the field u and we have:

$$E(x, y, z) = u(x, y, z)e^{-j\beta z}$$

Where β represents the average variation of E , it is called propagation constant

Substituting the approximated solution E into the propagation equation gives

$$-\frac{\partial^2 u}{\partial z^2} + 2j\beta \frac{\partial u}{\partial z} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + (k^2 - \beta^2)u$$

At this stage, we assume the slow variation of optical field in the propagation direction (SVEA) which requires

$$\left| \frac{\partial^2 u}{\partial z^2} \right| \ll \left| 2\beta \frac{\partial u}{\partial z} \right|$$

Ignoring the laplacian along z (Fresnel approximation) reduces the « paraxial » propagation equation to:

$$2i\beta \frac{\partial u}{\partial z} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + (k^2 - \beta^2)u$$

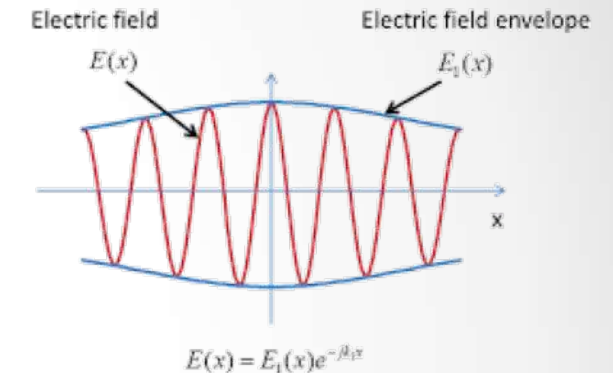
Note that it does not describe any polarization effects. The equation determines the evolution of the field for $z > 0$

$$\frac{\partial u}{\partial z} = \hat{D}u + \hat{W}u$$

With the operators on the right hand side being

$$\hat{D} = \frac{1}{2j\beta} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)$$

$$\hat{W} = \frac{1}{2j\beta} (k^2 - \beta^2)$$



$$\hat{D} = \frac{1}{2j\beta} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)$$

$$\hat{W} = \frac{1}{2j\beta} (k^2 - \beta^2)$$

Operator D represents free-space propagation (diffraction)
Operator W describes guiding effects

Assuming that operators are z-independent, the solution is symbolically written as

$$u(x, y, z + \Delta z) = e^{(\hat{D} + \hat{W})\Delta z} u(x, y, z)$$

And using the Baker-Hausdorf lemma $e^{\hat{A}}e^{\hat{B}} = e^{\hat{A} + \hat{B}} e^{\frac{1}{2}[\hat{A}, \hat{B}]}$

NB: $[A, B]$ is the commutator

Given that A and B each commutes with $[A, B]$. We can then approximate the amplitude by

$$u(x, y, z + \Delta z) \approx e^{\hat{D}\Delta z} e^{\hat{W}\Delta z} u(x, y, z)$$

Consider an equation containing only an operator D $2j\beta \frac{\partial u(x, y, z)}{\partial z} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) u(x, y, z)$

Define 2D continuous Fourier transform as

$$\tilde{u}(k_x, k_y, z) = \int_{-\infty}^{+\infty} u(x, y, z) e^{-j(k_x x + k_y y)} dx dy \equiv F_{x,y} \{u(x, y, z)\}$$

and its inverse

$$u(x, y, z) = \int_{-\infty}^{+\infty} \tilde{u}(k_x, k_y, z) e^{j(k_x x + k_y y)} dk_x dk_y F_{x,y}^{-1} \{\tilde{u}(x, y, z)\}$$

Apply the 2D FT to the top equation and have for Fourier components

$$2j\beta \frac{\partial}{\partial z} \tilde{u}(x, y, z) = -(k_x^2 + k_y^2) \tilde{u}(x, y, z)$$

Integrate the above from z to $z + \Delta z$ and have

$$\begin{aligned}\tilde{u}(x, y, z + \Delta z) &= e^{-\frac{1}{2j\beta}(k_x^2 + k_y^2)\Delta z} \tilde{u}(x, y, z) \\ &\equiv \hat{H}(k_x, k_y, \Delta z) \tilde{u}(x, y, z)\end{aligned}$$

To analyse operator $\hat{W}$, introduce the relation $\beta = k_0 n_{eff}$

Write also refractive index n as $\bar{n} = n_{eff} + \Delta\bar{n}$

Substituting the above into previous equation gives an expression for operator $\hat{W}$ to the first order in n

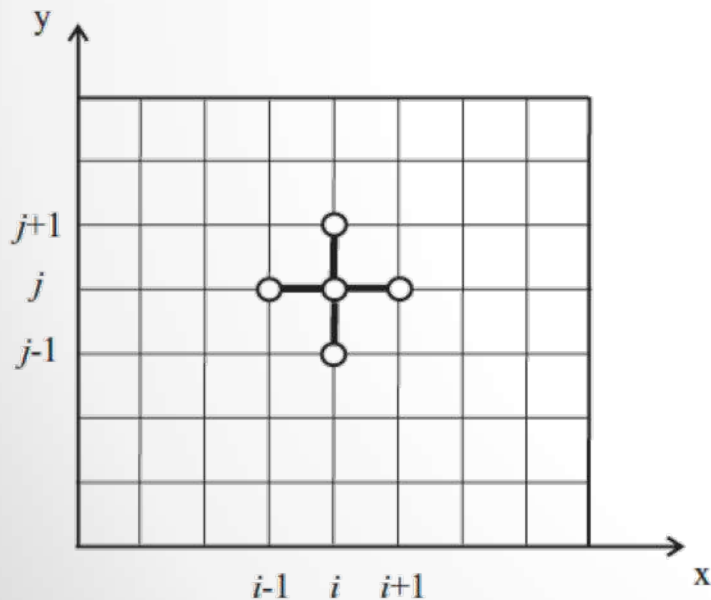
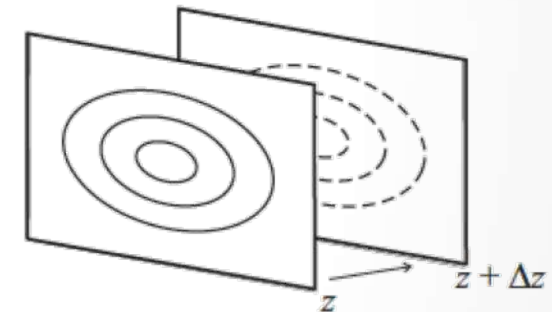
$$\hat{W} = -jk_0 \Delta n$$

II Implementation of the BPM method

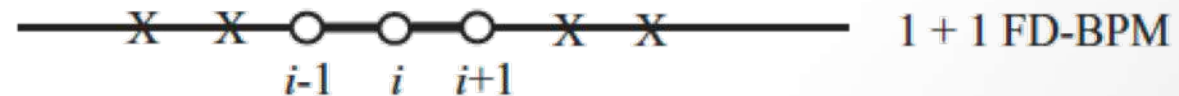
We will use the Fourier Transform split-step method

A general scheme of FD-BPM is described by the propagator U , which we use to advance field as

$$\tilde{\mathbf{E}}_t(z + \Delta z) = U(\Delta z) \tilde{\mathbf{E}}_t(z)$$

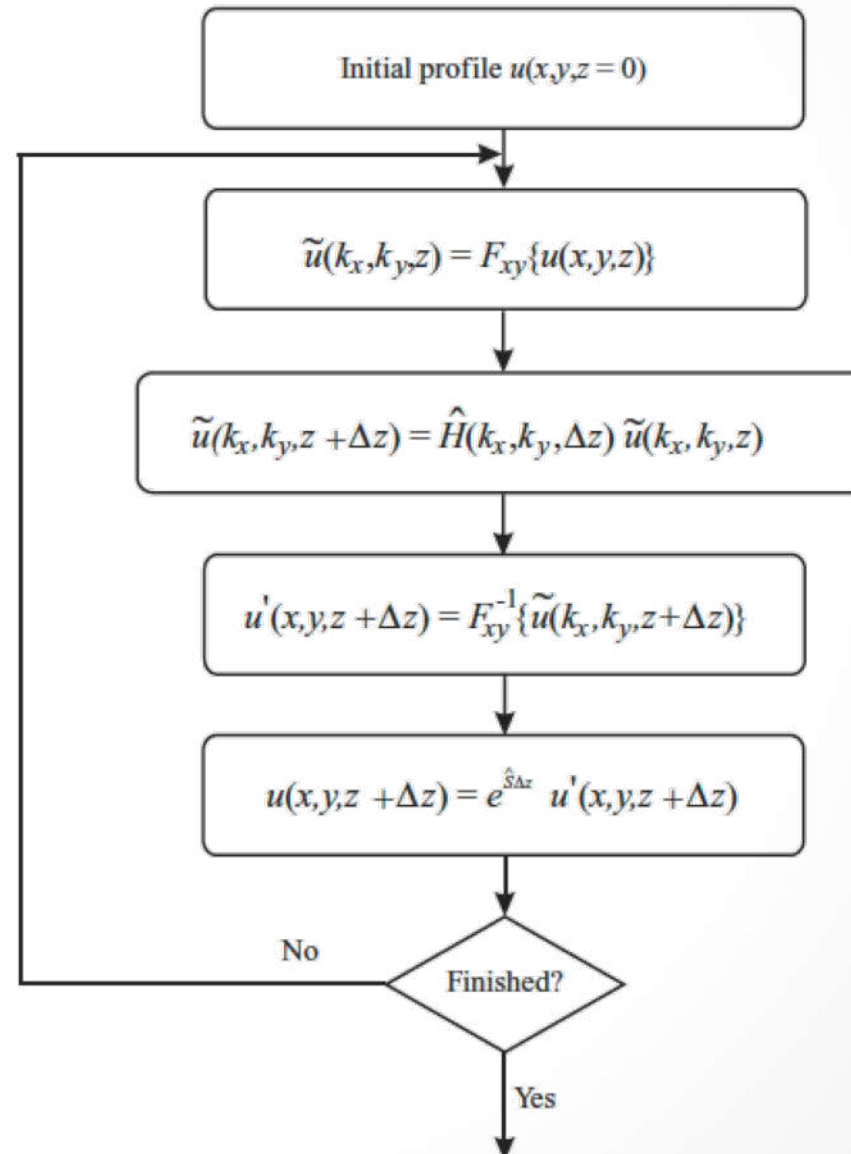


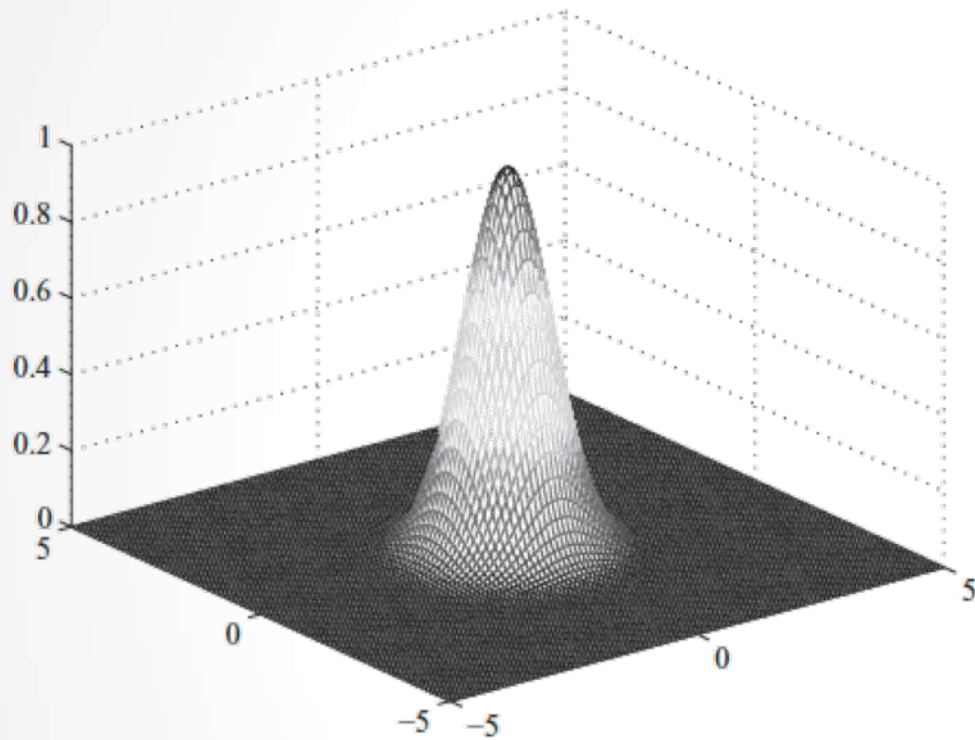
1 + 2 FD-BPM



1 + 1 FD-BPM

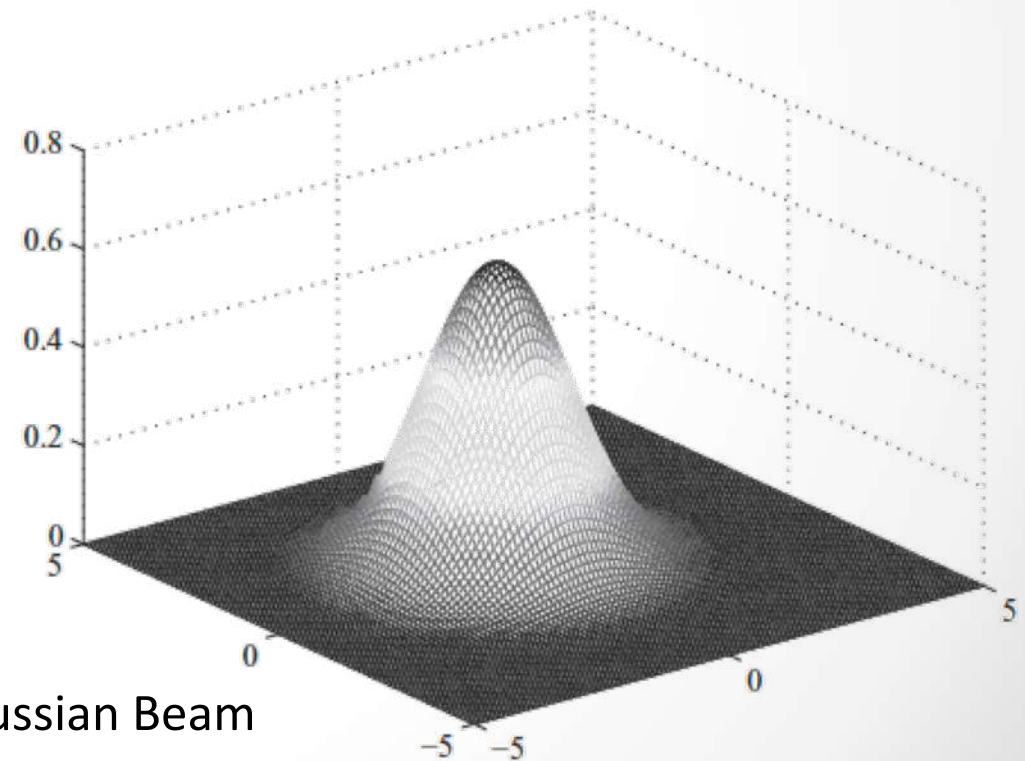
Flow diagram for the BPM split-step method





Initial Gaussian Beam

Pure diffraction process



Propagated Gaussian Beam

```
% File name: Gauss.m
% Propagation of 2D Gaussian pulse by paraxial FT split-step BPM
clear all
close all
N= 10; delta_x = 1/N; minmax=10;
x = -minmax:delta_x:minmax; % creation of space arguments
y = -minmax:delta_x:minmax; % creation of space arguments
delta_z = 5.0; % step size along z-axis
delta_n = 0.4; k_zero = 100;
beta = 20; % propagation constant
%
k_x = -minmax:1/N:minmax; % creation of Fourier variables
k_y = -minmax:1/N:minmax; % creation of Fourier variables
%
%
u_init=(exp(-x.^2))*exp(-y.^2); % initial Gaussian pulse
mesh(x,y,abs(u_init)); % plots original pulse
```

```
temp = delta_z/(2*beta);
H_transfer = (exp(1i*temp*k_x.^2))*(exp(1i*temp*k_y.^2));
H_transfer = fftshift(H_transfer); S_phase = exp(-1i*k_zero*delta_n);
%
for n=1:100
    z = fft2(u_init);
    zz = z.*H_transfer;
    u_prime = ifft2(zz);
    u = S_phase.*u_prime;
    u_init = u;
    mesh(x,y,abs(u_init)) % plots pulse after propagation
    Zlim=1;
    pause(0.1)
end
%
%close all
```

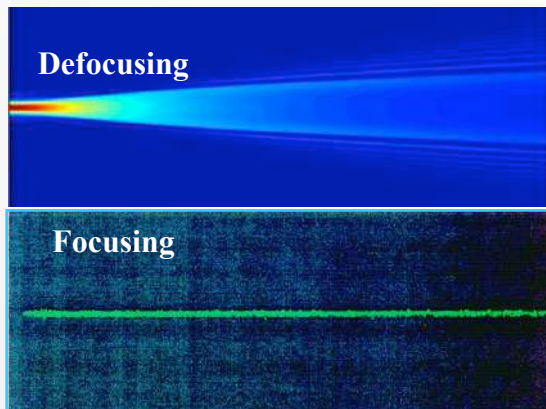
Nonlinear wave equation

$$\nabla^2 E = \frac{1}{c^2} \frac{\partial^2 D}{\partial t^2}$$

Displacement field is given by

$$D = \varepsilon E = n^2 E = (n + \Delta n)^2 E \approx (n + 2n\Delta n)E$$

We obtain the nonlinear Schrödinger equation



$$i \frac{\partial \psi}{\partial z} - \frac{1}{2k} \nabla_{\perp}^2 \psi - \frac{k}{n_0} n_2 |\psi|^2 \psi = 0$$

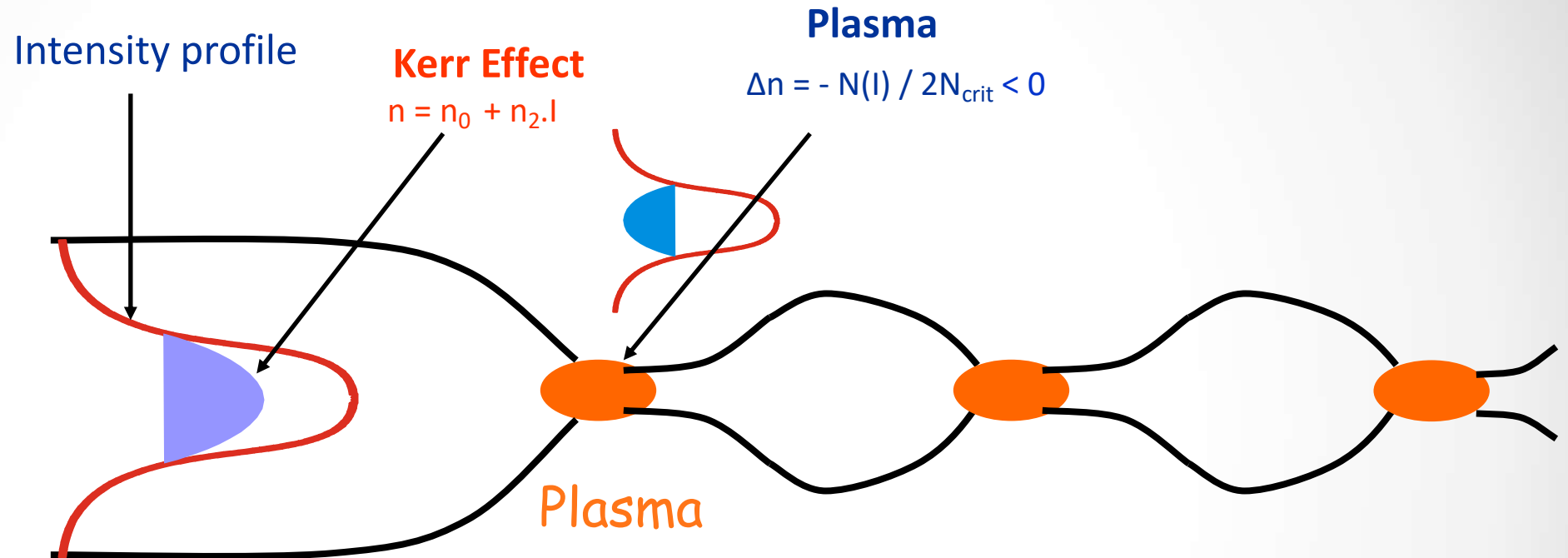
Propagation

Diffraction

Non-linéarity

Kerr Coefficient

Intensity



Linear part

$$i \frac{\partial \psi}{\partial z} = \frac{1}{2k} \frac{\partial^2 \psi}{\partial x^2}$$

Nonlinear part

$$i \frac{\partial \psi}{\partial z} = \frac{k}{n} n_2 |\psi|^2 \psi$$

