

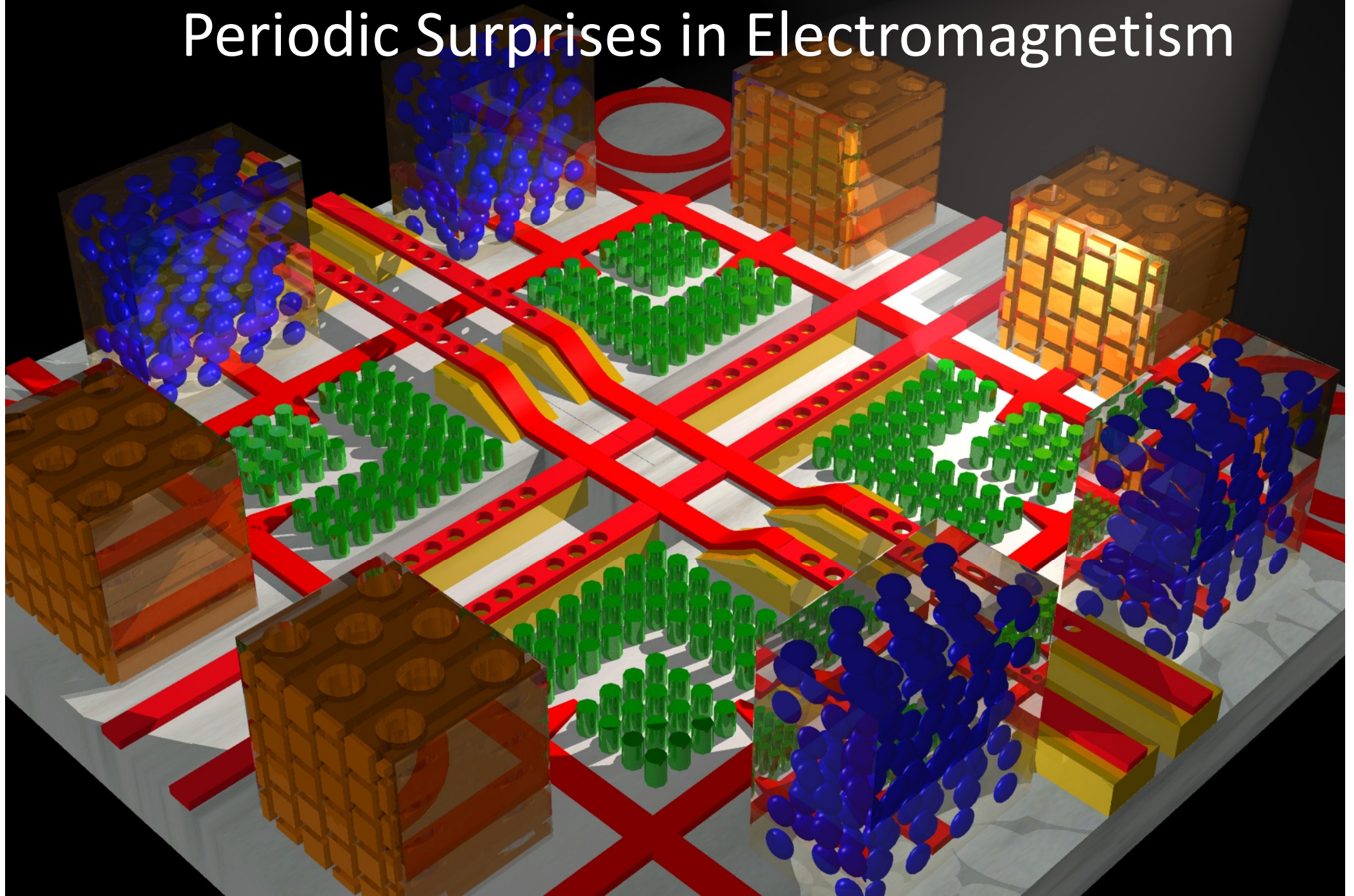
Electromagnetic modeling of photonic crystals

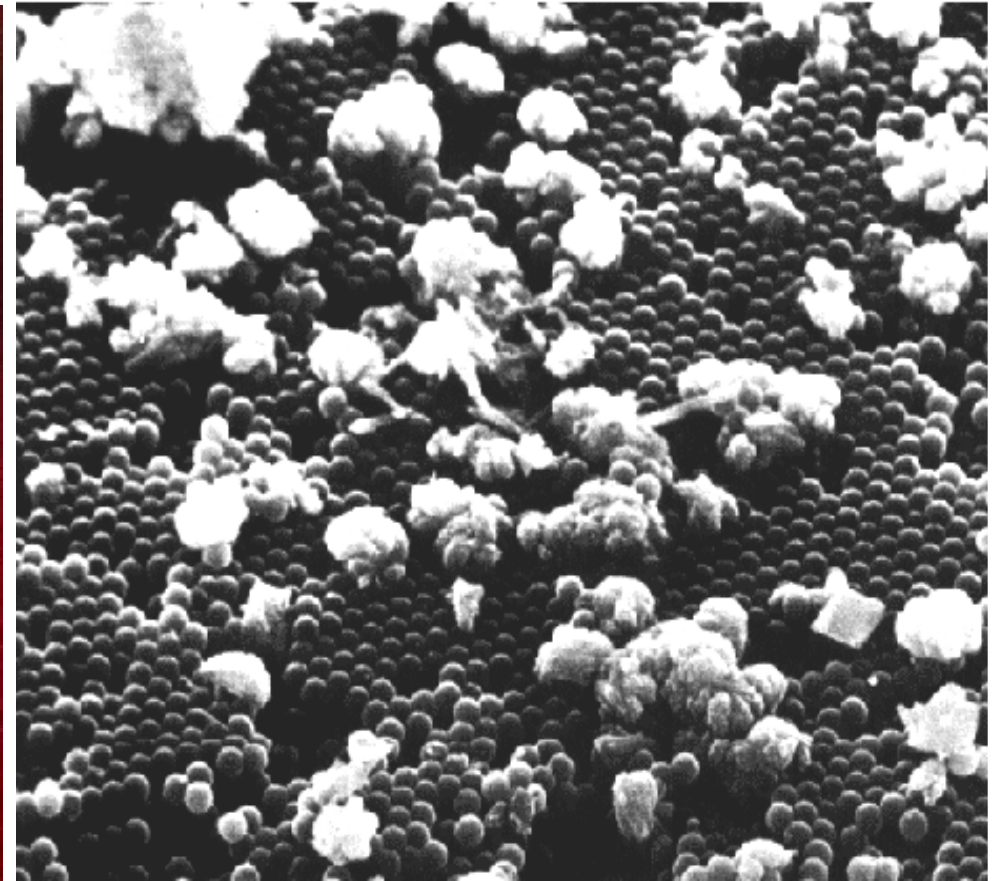
Jean-Philippe Colombier

Jean.philippe.colombier@univ-st-etienne.fr

Photonic Crystals:

Periodic Surprises in Electromagnetism



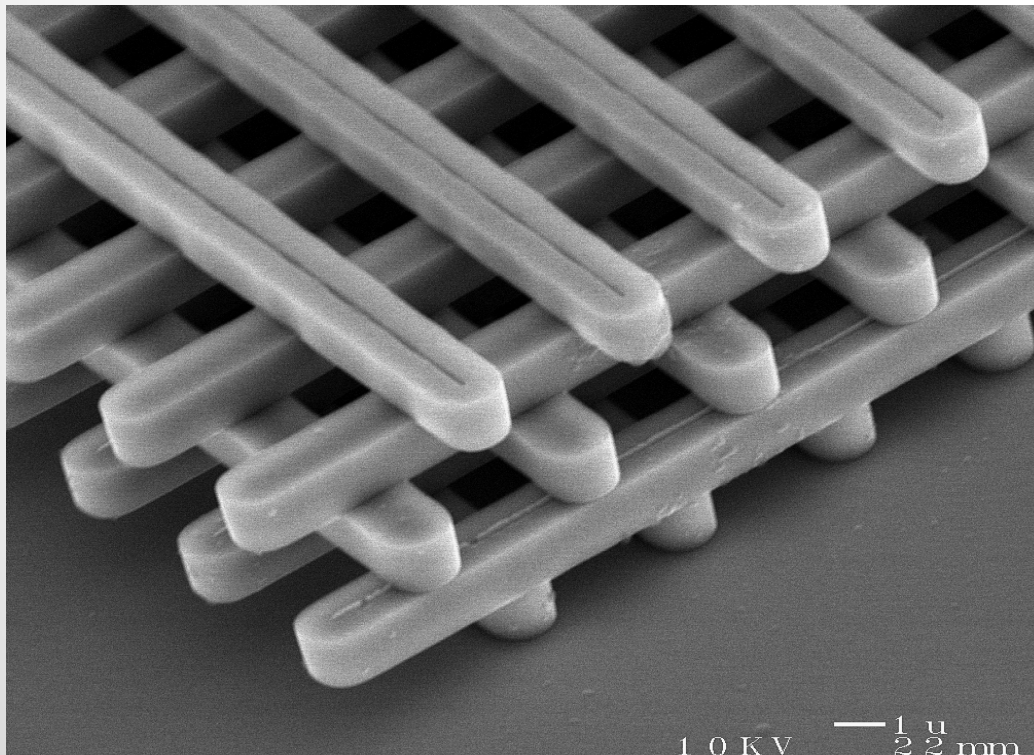


Periodic structure → striking colour effect even in the absence of pigments

Requirement: overlapping of frequency gaps along different directions

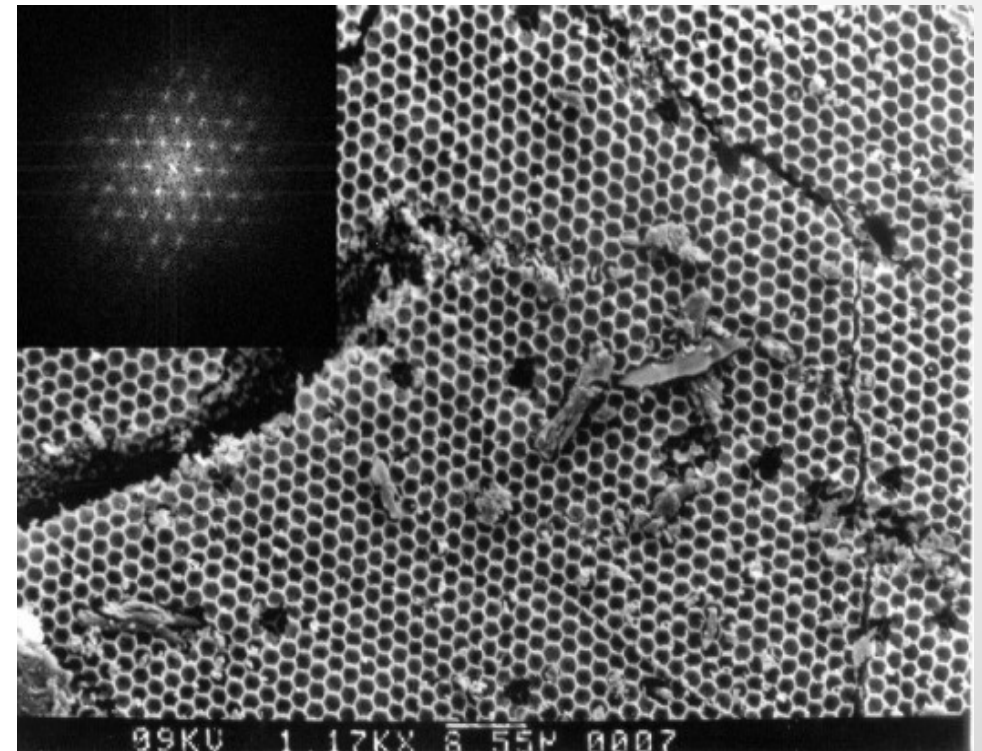
- High ratio of dielectric indices
- Same average optical path in different media
- Dielectric networks should be connected

Woodpile structure



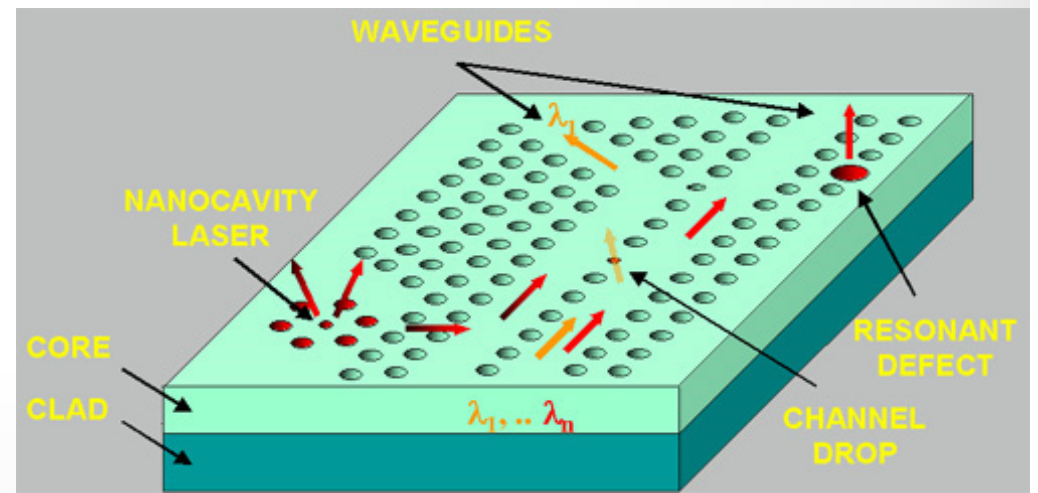
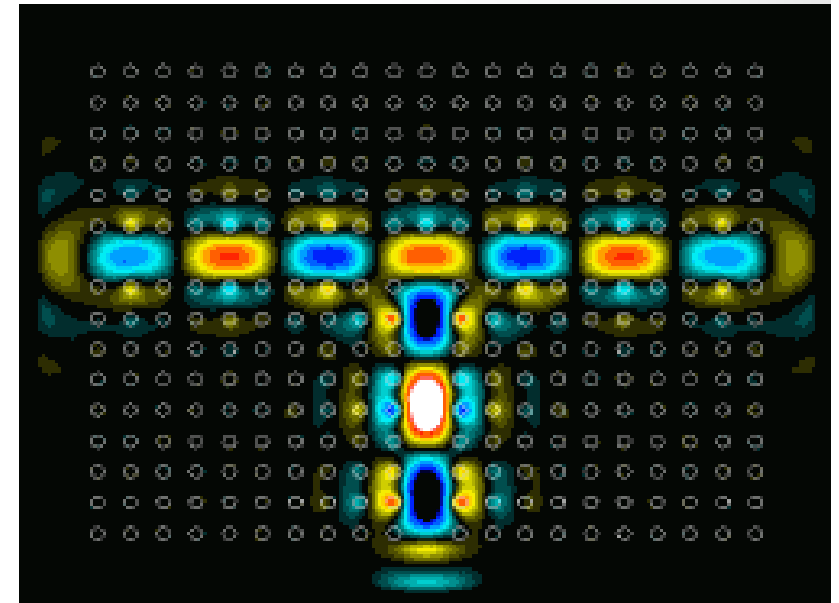
S. Lin et al., Nature (1998)

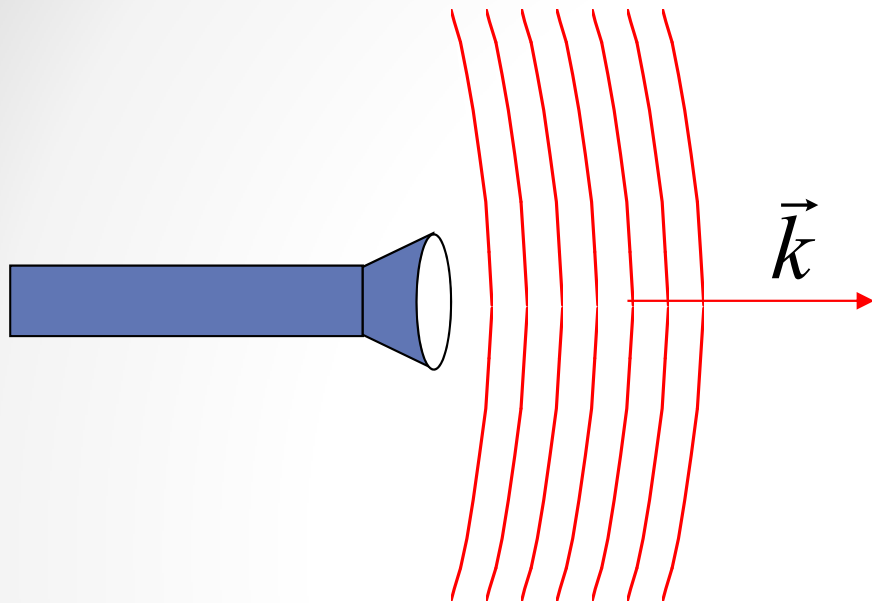
Inverted Opals



J. Wijnhoven & W. Vos, Science (1998)

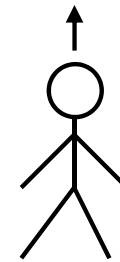
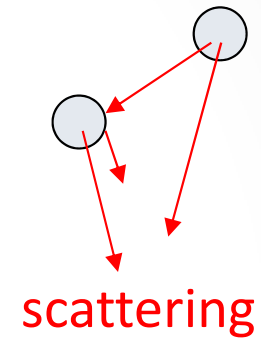
- Photonic Crystals
 - complex dielectric environment that controls the flow of radiation
 - designer vacuum for the emission and absorption of radiation
- Passive devices
 - dielectric mirrors for antennas
 - micro-resonators and waveguides
- Active devices
 - low-threshold nonlinear devices
 - microlasers and amplifiers
 - efficient thermal sources of light
- Integrated optics
 - controlled miniaturisation
 - pulse sculpturing

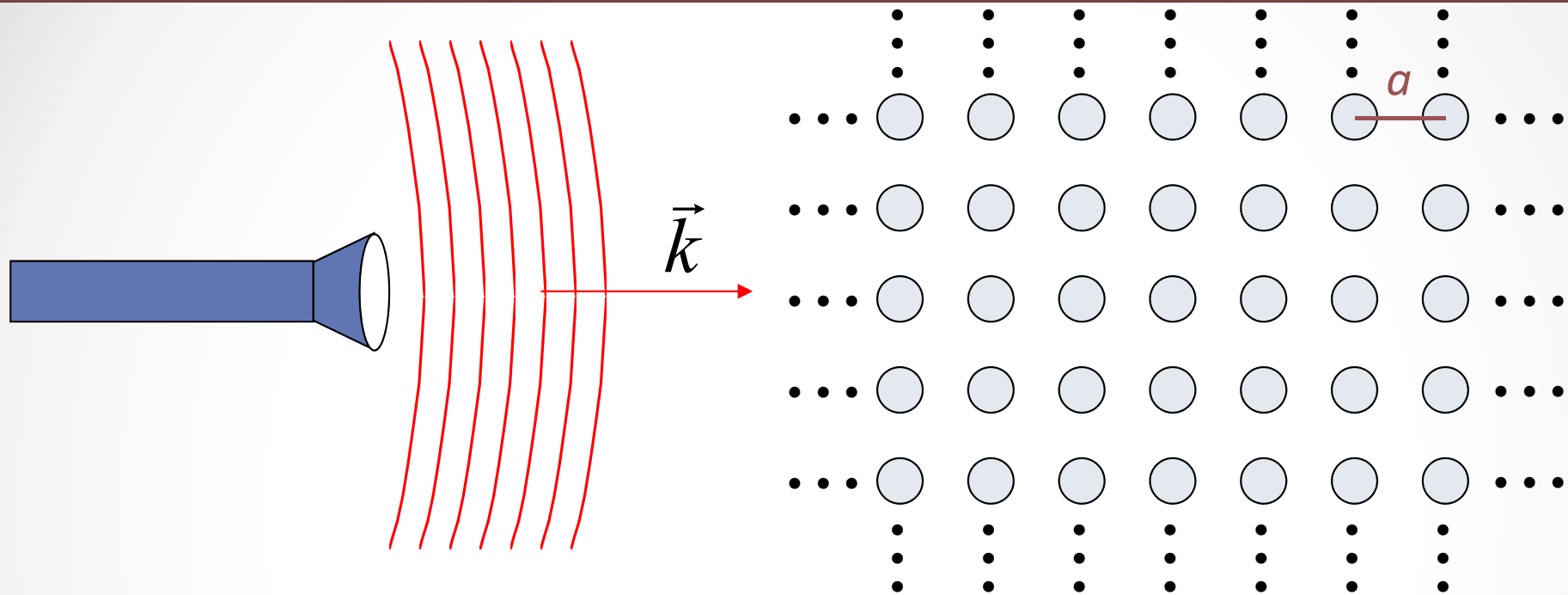




planewave

$$\vec{E}, \vec{H} \sim e^{i(\vec{k} \cdot \vec{x} - \omega t)}$$
$$|\vec{k}| = \omega / c = \frac{2\pi}{\lambda}$$





planewave

$$\vec{E}, \vec{H} \sim e^{i(\vec{k} \cdot \vec{x} - \omega t)}$$

$$|\vec{k}| = \omega / c = \frac{2\pi}{\lambda}$$

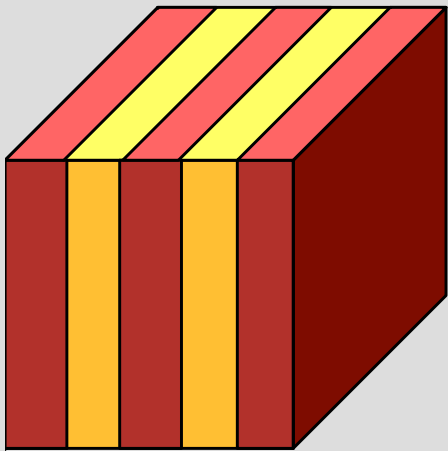
for **most** λ , beam(s) propagate
through crystal **without scattering**
(scattering cancels **coherently**)

...but for **some** λ ($\sim 2a$), no light can propagate: **a photonic band gap**

periodic electromagnetic media

1887

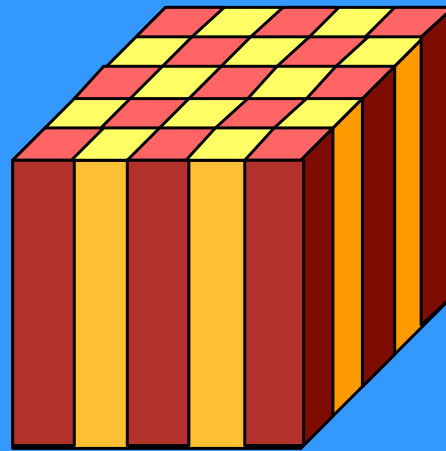
1-D



periodic in
one direction

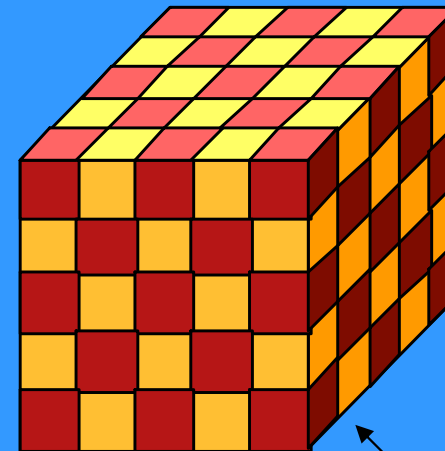
1987

2-D



periodic in
two directions

3-D

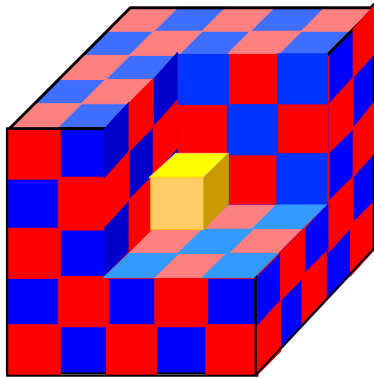


periodic in
three directions

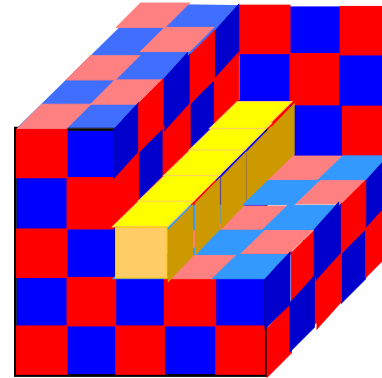
(need a
more
complex
topology)

with photonic band gaps: “optical insulators”

periodic electromagnetic media



can trap light in **cavities**

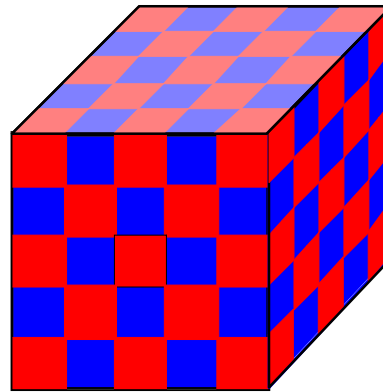


and **waveguides** ("wires")

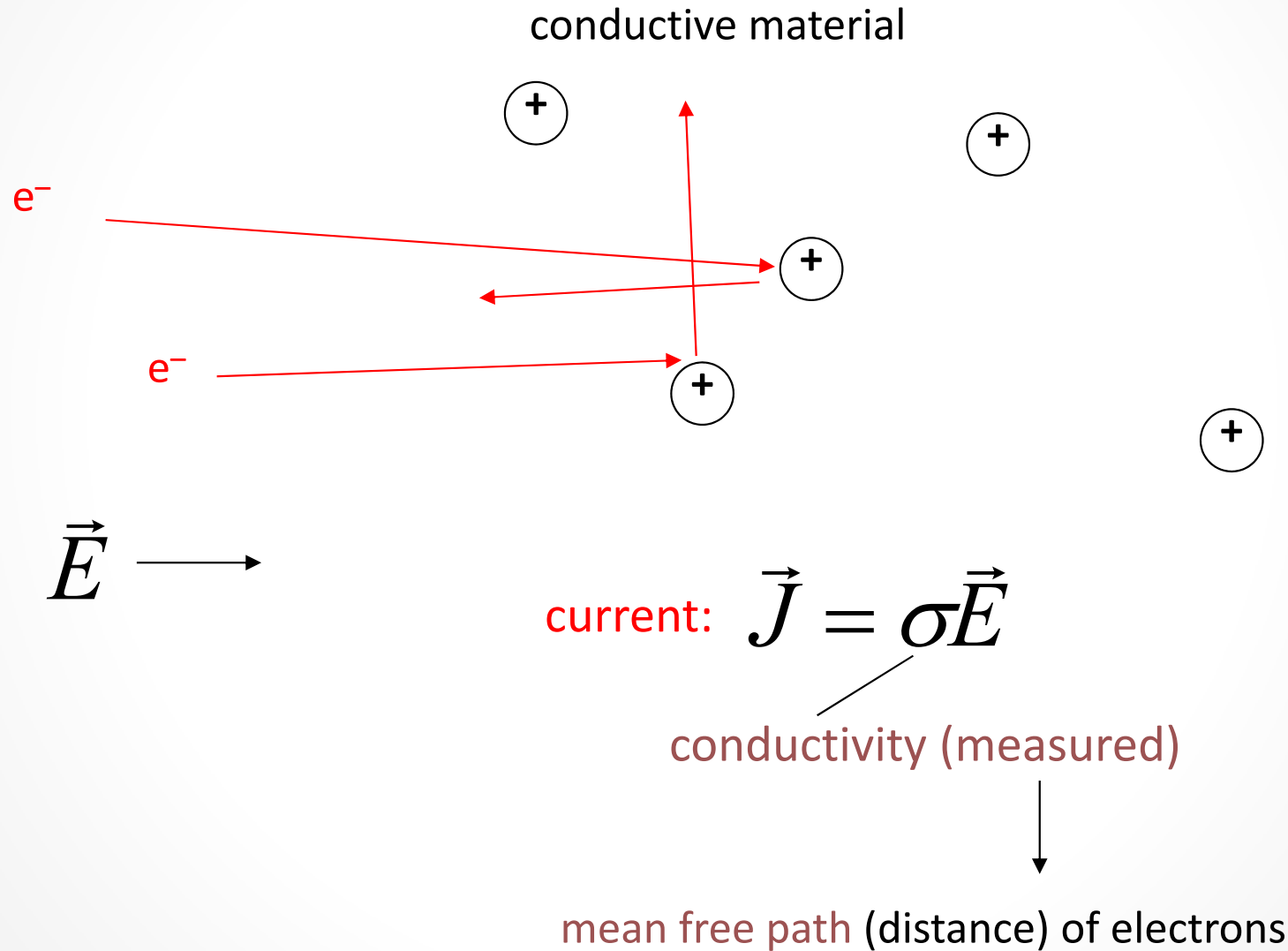
magical oven mitts for
holding and controlling light

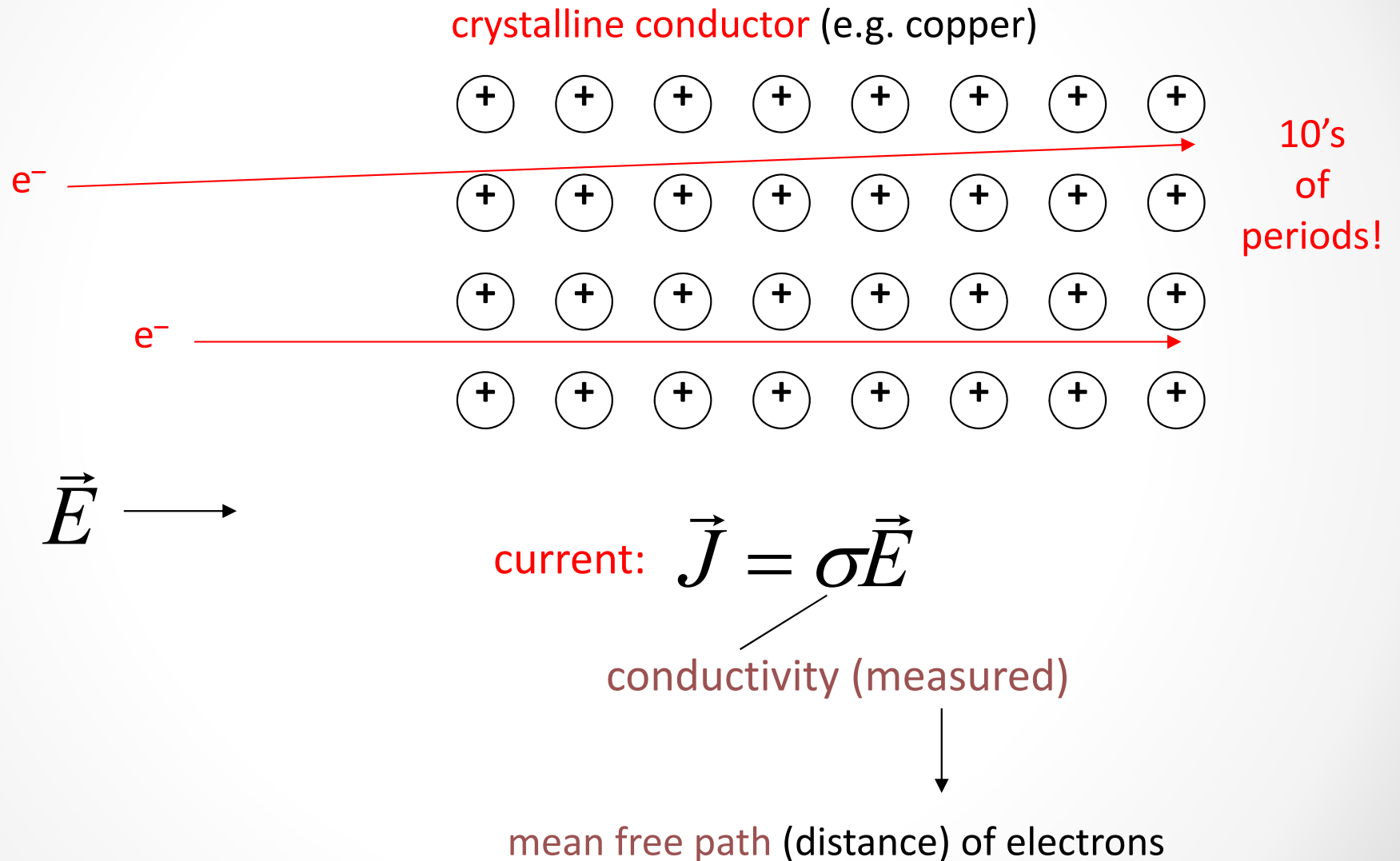
with photonic band gaps: "**optical insulators**"

periodic electromagnetic media



But how can we **understand** such complex systems?
Add up the infinite sum of scattering? Ugh!

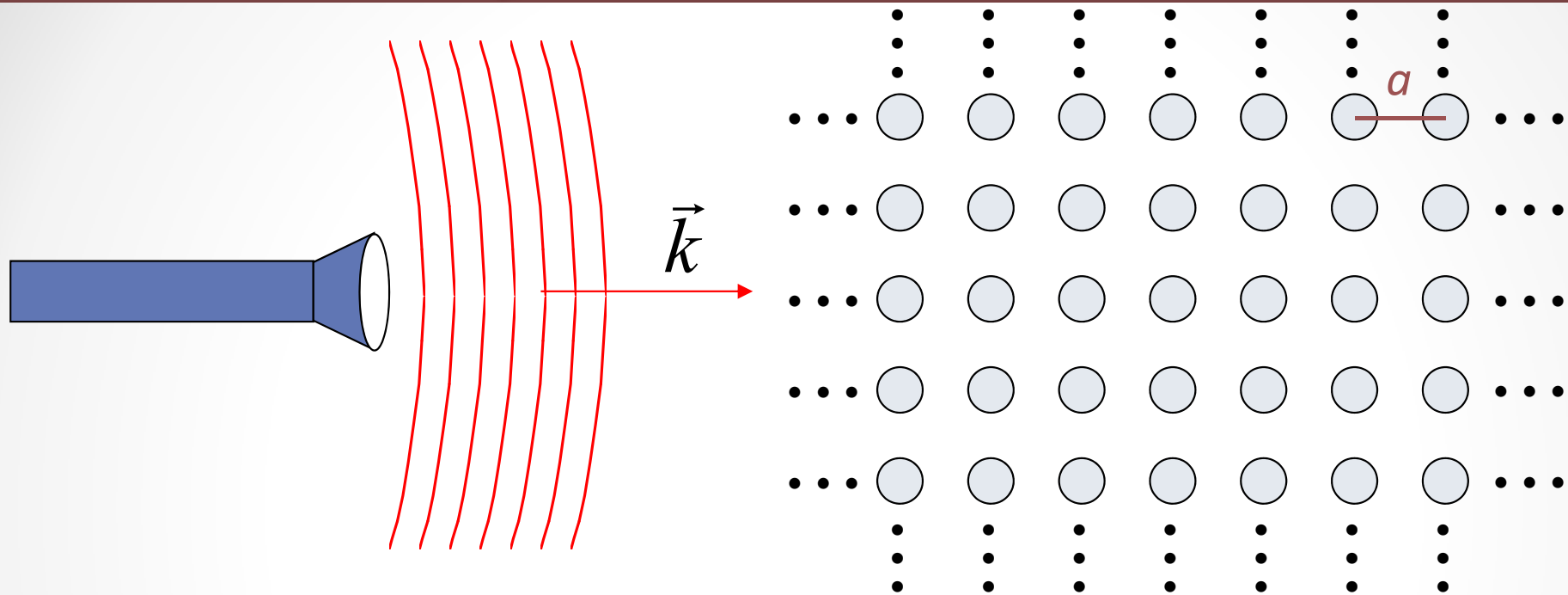




- 1 electrons are **waves** (quantum mechanics)
- 2 waves in a **periodic medium** can propagate **without scattering**:

Bloch's Theorem (1d: Floquet's)

The foundations **do not depend on the specific wave equation**.



planewave

$$\vec{E}, \vec{H} \sim e^{i(\vec{k} \cdot \vec{x} - \omega t)}$$

$$|\vec{k}| = \omega / c = \frac{2\pi}{\lambda}$$

for **most** λ , beam(s) propagate
through crystal **without scattering**
(scattering cancels **coherently**)

...but for **some** λ ($\sim 2a$), no light can propagate: a **photonic band gap**

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial}{\partial t} \vec{H} = i \frac{\omega}{c} \vec{H}$$

First task:

get rid of this mess

$$\vec{\nabla} \times \vec{H} = \varepsilon \frac{1}{c} \frac{\partial}{\partial t} \vec{E} + \cancel{\vec{J}}^0 = i \frac{\omega}{c} \varepsilon \vec{E}$$

dielectric function $\varepsilon(\mathbf{x}) = n^2(\mathbf{x})$

$$\nabla \times \frac{1}{\varepsilon} \nabla \times \vec{H} = \left(\frac{\omega}{c} \right)^2 \vec{H}$$

+ constraint

$$\nabla \cdot \vec{H} = 0$$

eigen-operator

eigen-value

eigen-state

$$\underbrace{\nabla \times \frac{1}{\varepsilon} \nabla \times \vec{H}}_{\text{eigen-operator}} = \underbrace{\left(\frac{\omega}{c} \right)^2}_{\text{eigen-value}} \underbrace{\vec{H}}_{\text{eigen-state}} \quad \begin{array}{l} \text{+ constraint} \\ \nabla \cdot \vec{H} = 0 \end{array}$$

Hermitian for real (lossless) ε

➔ well-known properties from linear algebra:

ω are real (lossless)

eigen-states are orthogonal

eigen-states are complete (give all solutions)

[G. Floquet, “Sur les équations différentielles linéaires à coefficients périodiques,” *Ann. École Norm. Sup.* **12**, 47–88 (1883).]
 [F. Bloch, “Über die quantenmechanik der electronen in kristallgittern,” *Z. Physik* **52**, 555–600 (1928).]

if eigen-operator is periodic, then Bloch-Floquet theorem applies:

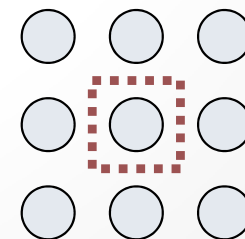
can choose:

$$\vec{H}(\vec{x}, t) = e^{i(\vec{k} \cdot \vec{x} - \omega t)} \vec{H}_{\vec{k}}(\vec{x})$$

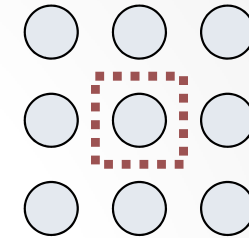
planewave
periodic “envelope”

Corollary 1: $\mathbf{k}$ is conserved, i.e. no scattering of Bloch wave

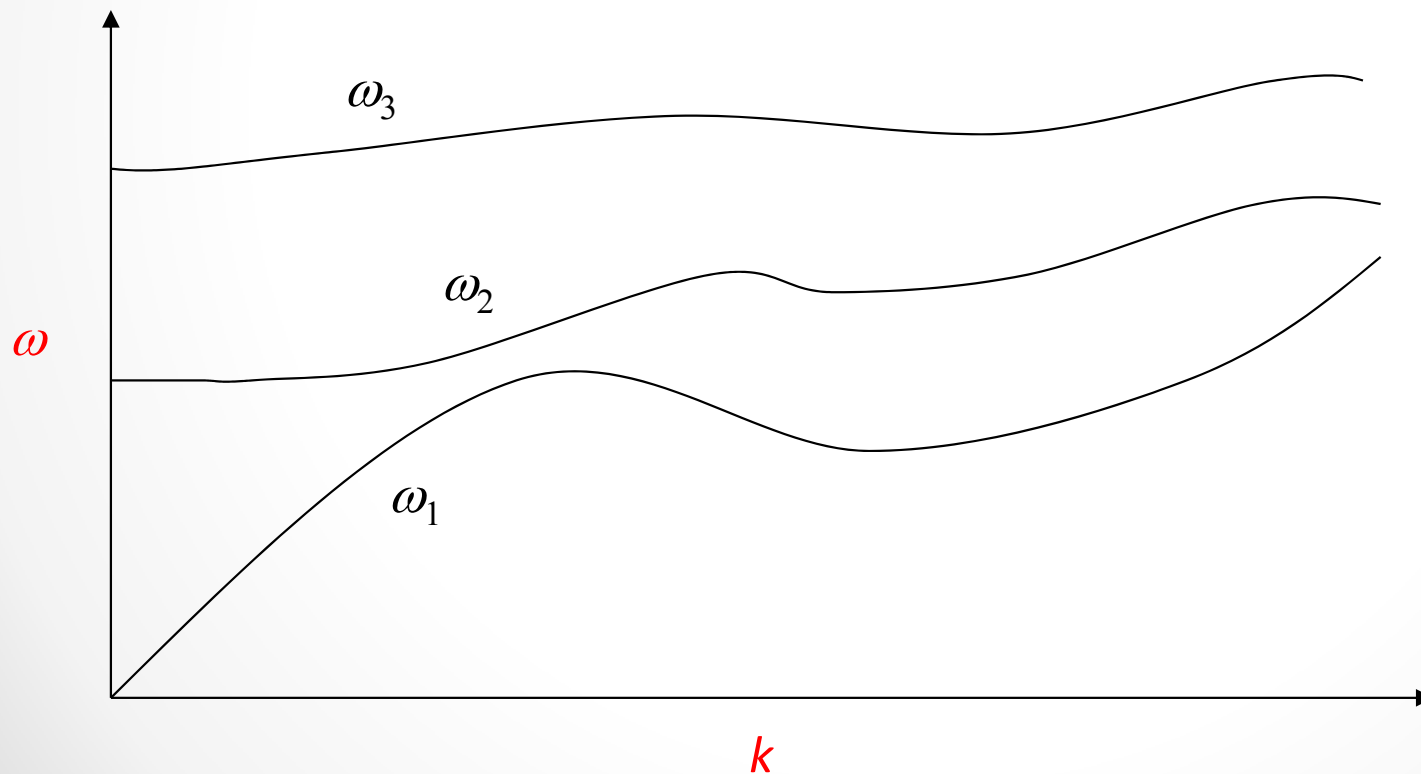
Corollary 2: $\vec{H}_{\vec{k}}$ given by finite unit cell,
 so ω are discrete $\omega_n(\mathbf{k})$



Corollary 2: $\vec{H}_{\vec{k}}$ given by finite unit cell,
so ω are discrete $\omega_n(\mathbf{k})$



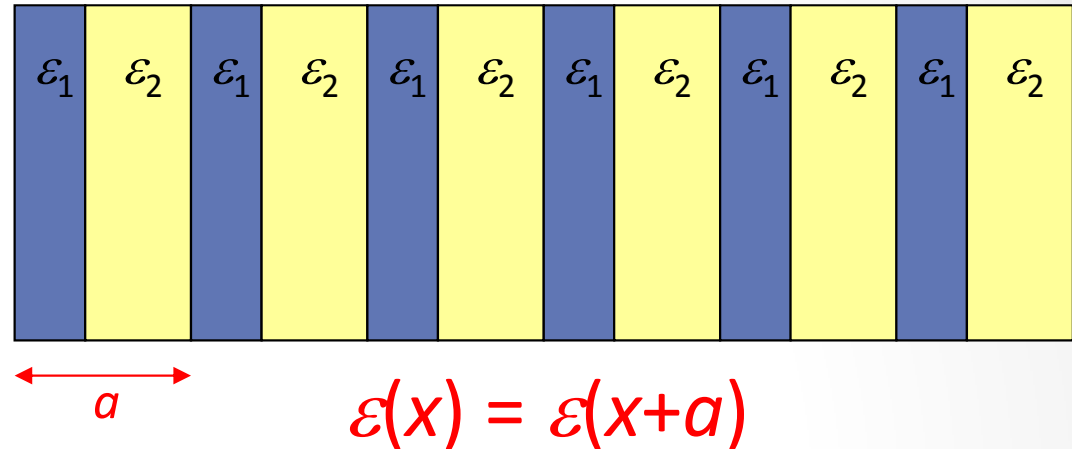
band diagram (dispersion relation)



map of
what states
exist &
can interact

?
range of k ?

$$H(x) = e^{ikx} H_k(x)$$



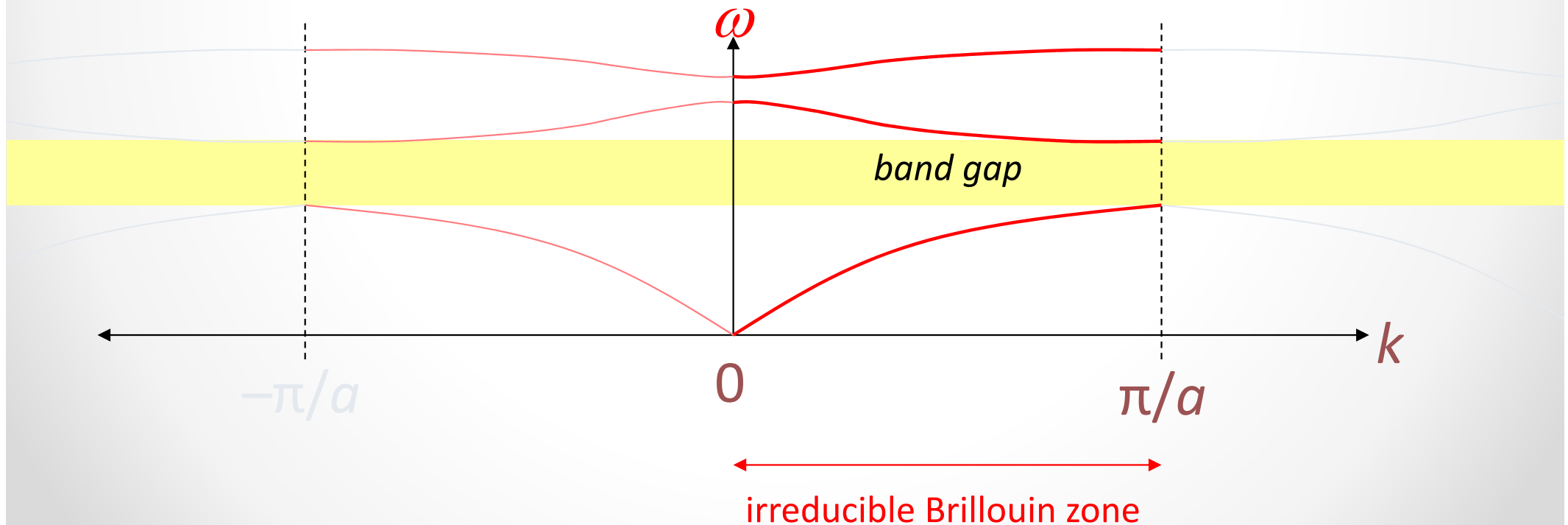
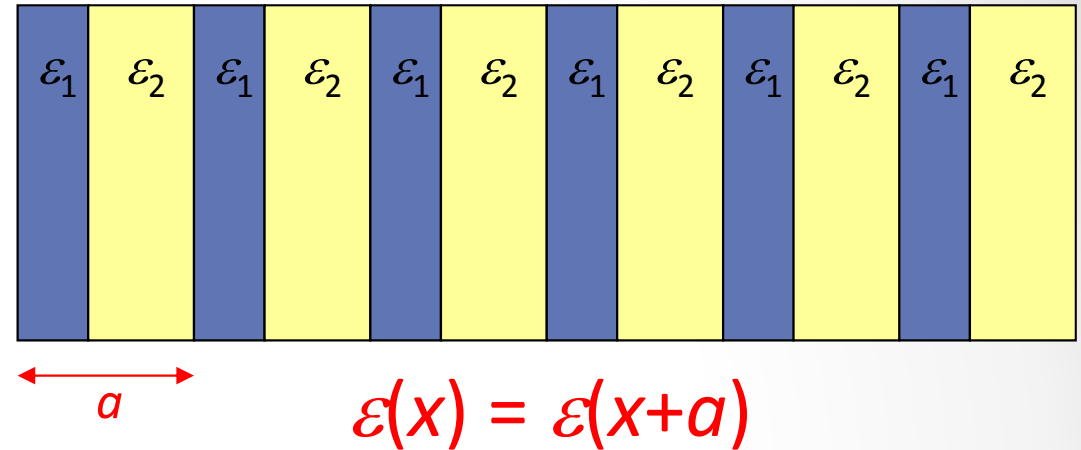
Consider $k+2\pi/a$:
$$e^{i(k+\frac{2\pi}{a})x} H_{k+\frac{2\pi}{a}}(x) = e^{ikx} \left[e^{i\frac{2\pi}{a}x} H_{k+\frac{2\pi}{a}}(x) \right]$$

k is periodic:
 $k + 2\pi/a$ equivalent to k
 “quasi-phase-matching”

periodic!
 satisfies same
 equation as H_k
 $= H_k$

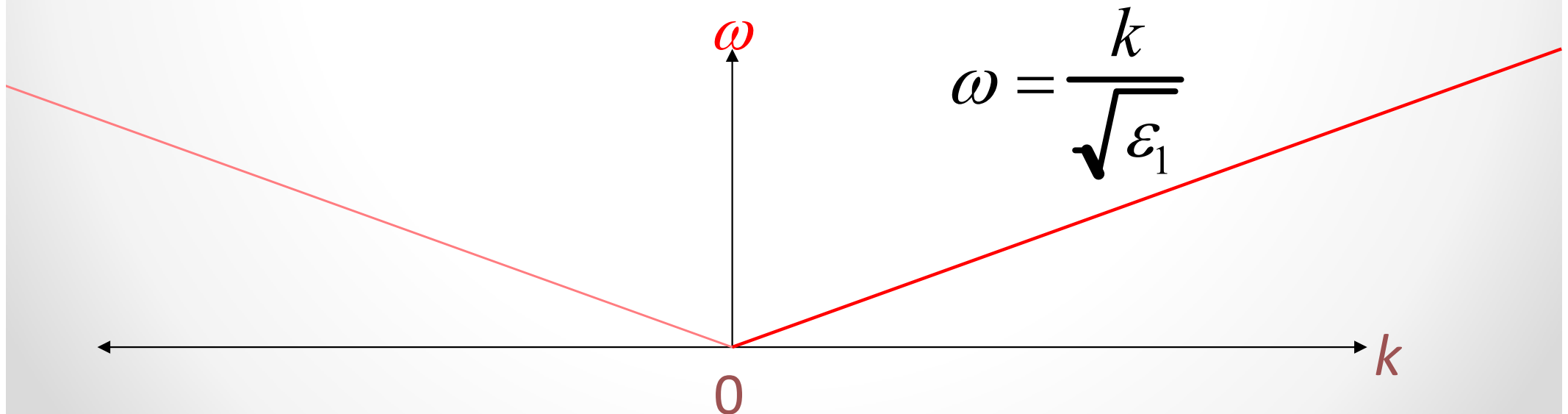
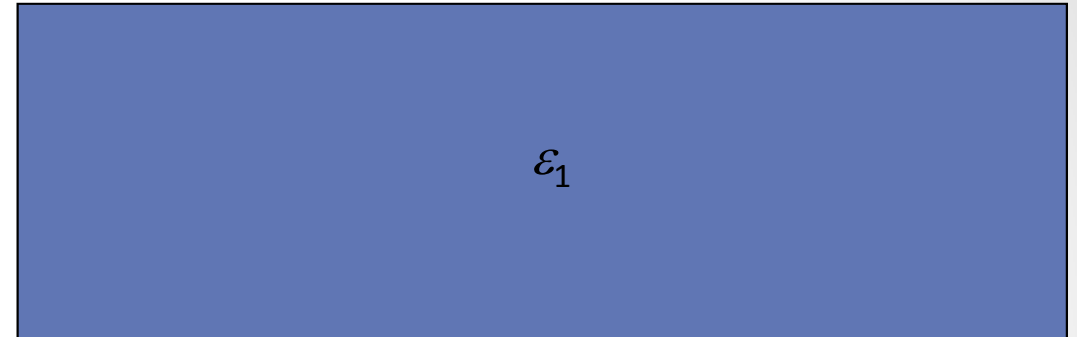
k is periodic:

$k + 2\pi/a$ equivalent to k
“quasi-phase-matching”



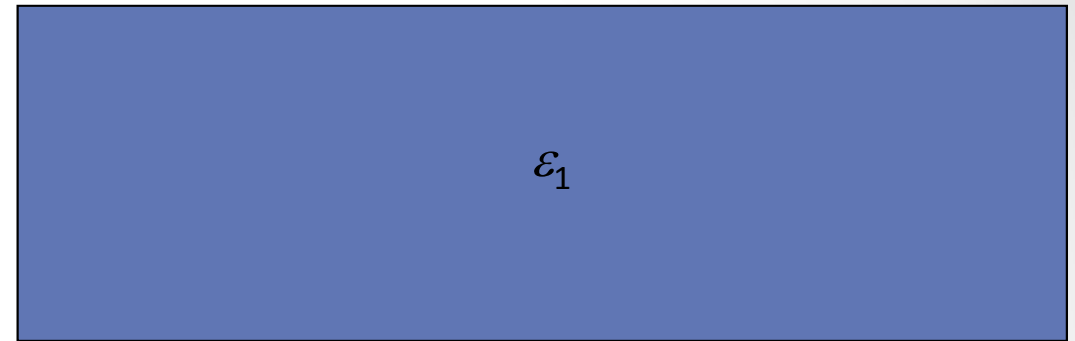
[Lord Rayleigh, “On the maintenance of vibrations by forces of double frequency, and on the propagation of waves through a medium endowed with a periodic structure,” *Philosophical Magazine* **24**, 145–159 (1887).]

Start with
a uniform (1d) medium:



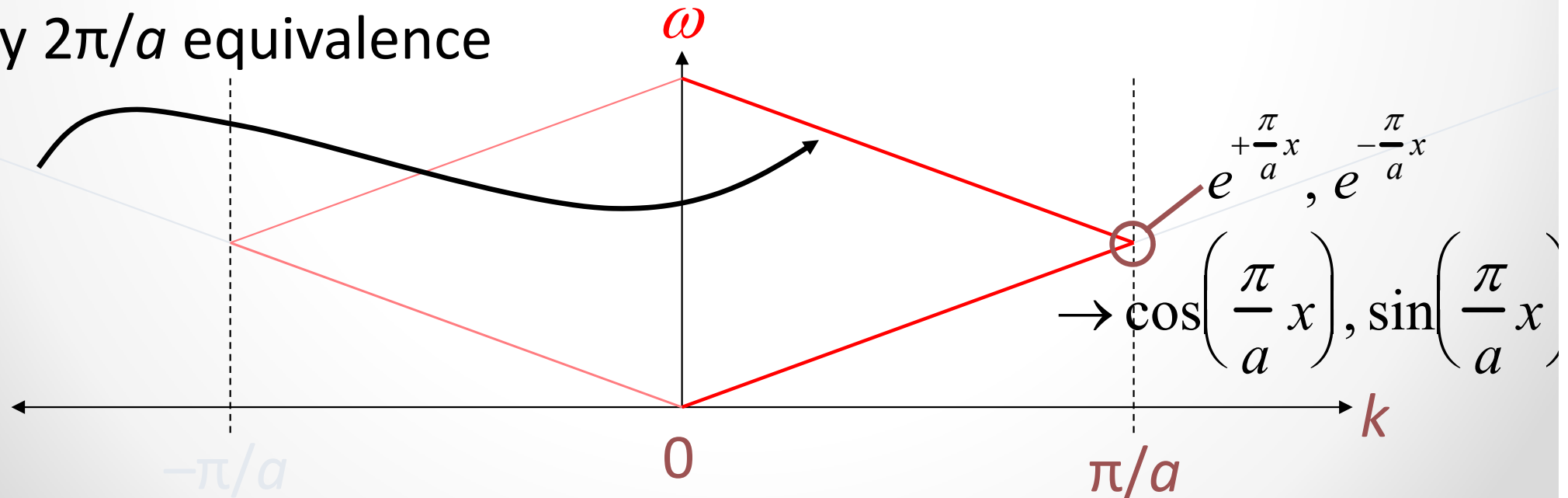
[Lord Rayleigh, “On the maintenance of vibrations by forces of double frequency, and on the propagation of waves through a medium endowed with a periodic structure,” *Philosophical Magazine* **24**, 145–159 (1887).]

Treat it as
“artificially” periodic



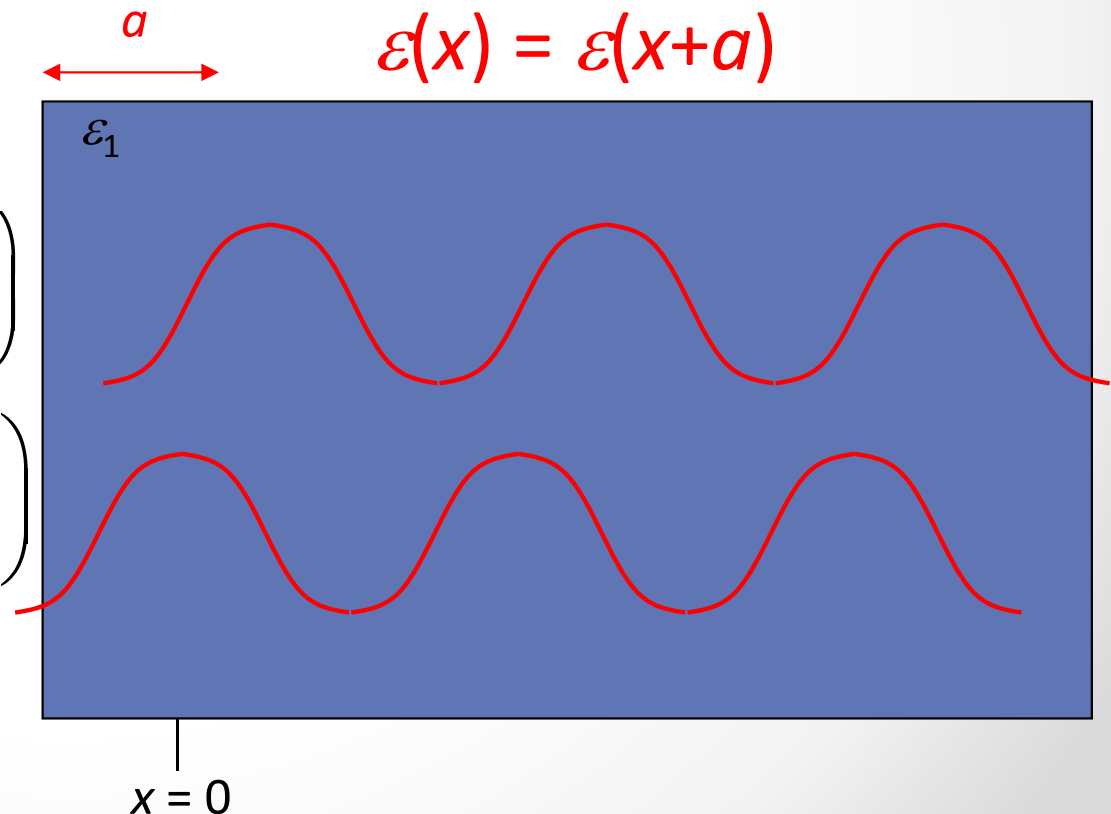
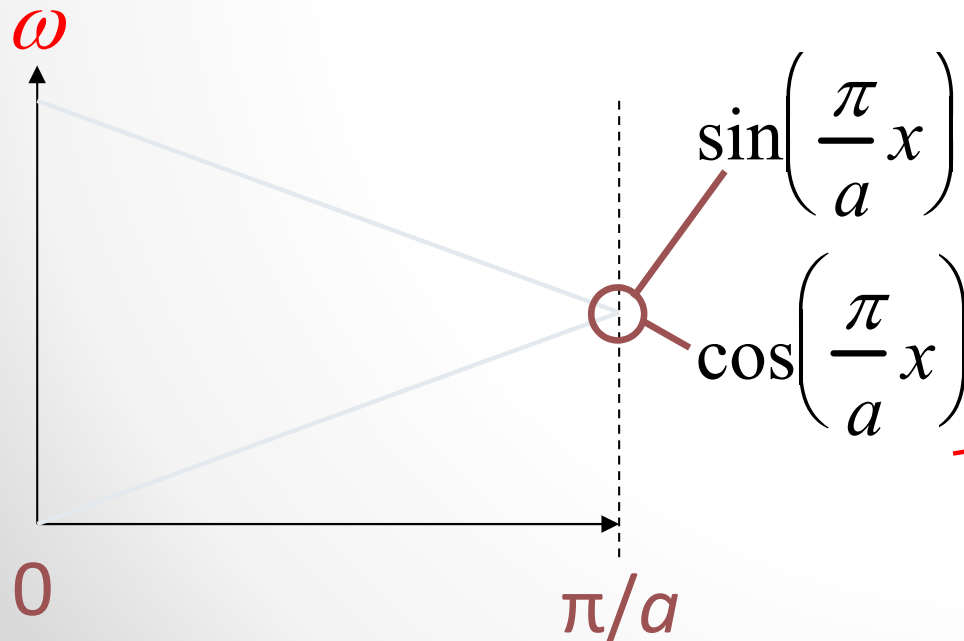
$$\epsilon(x) = \epsilon(x+a)$$

bands are “folded”
by $2\pi/a$ equivalence



[Lord Rayleigh, “On the maintenance of vibrations by forces of double frequency, and on the propagation of waves through a medium endowed with a periodic structure,” *Philosophical Magazine* **24**, 145–159 (1887).]

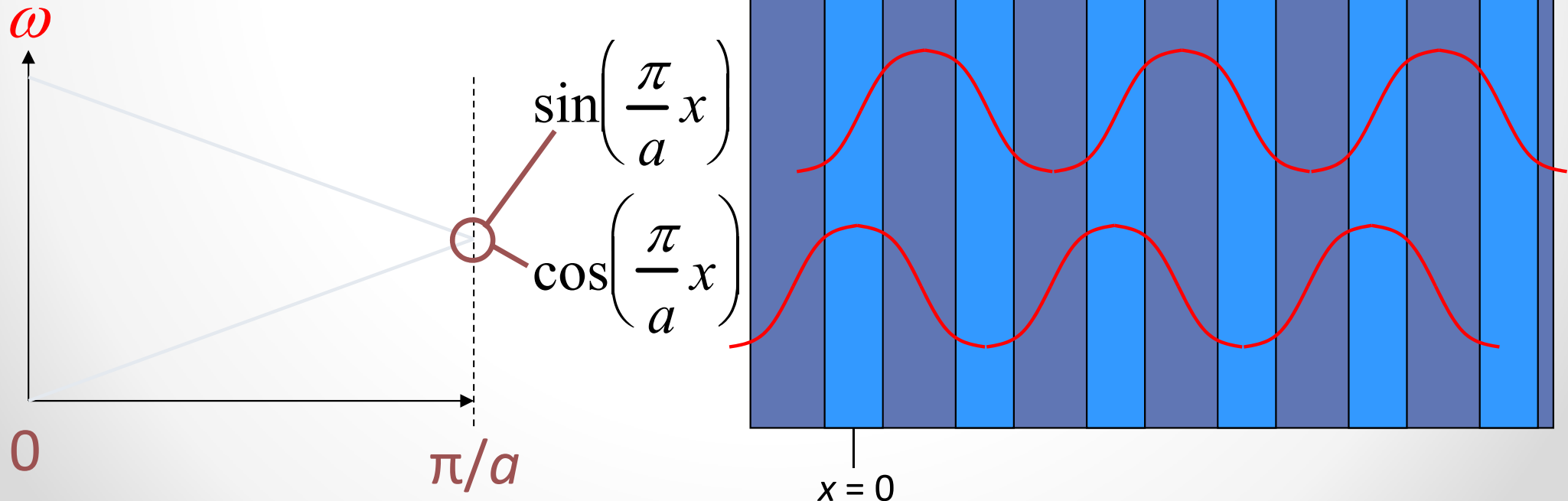
Treat it as
“artificially” periodic



[Lord Rayleigh, “On the maintenance of vibrations by forces of double frequency, and on the propagation of waves through a medium endowed with a periodic structure,” *Philosophical Magazine* **24**, 145–159 (1887).]

Add a small
“real” periodicity

$$\varepsilon_2 = \varepsilon_1 + \Delta\varepsilon$$



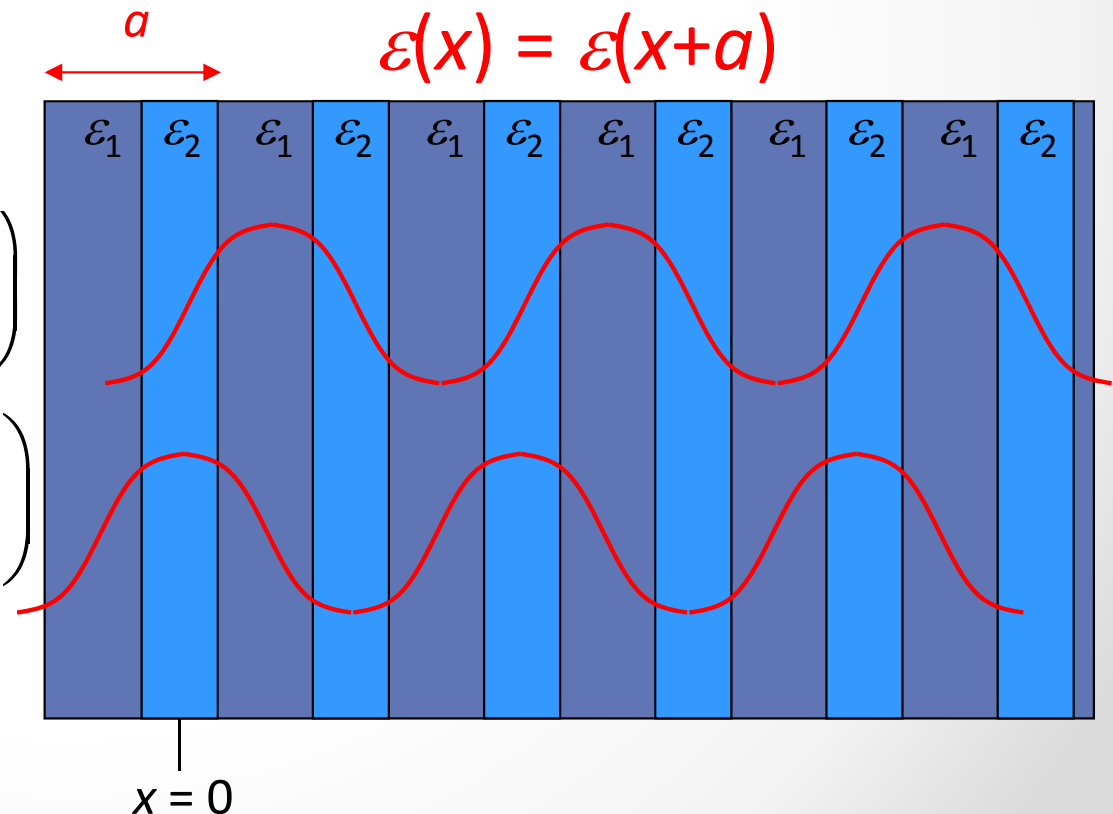
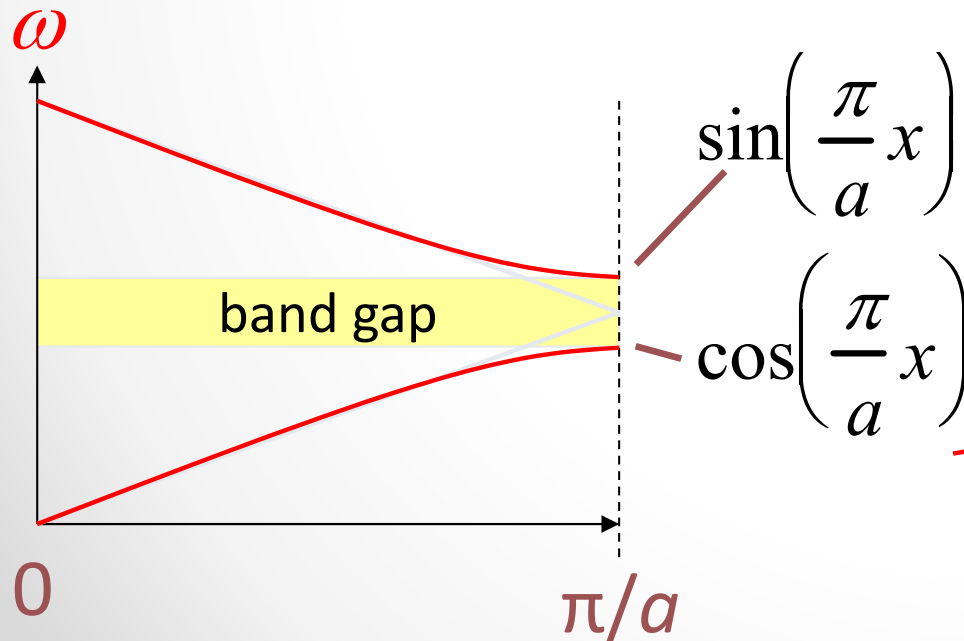
[Lord Rayleigh, “On the maintenance of vibrations by forces of double frequency, and on the propagation of waves through a medium endowed with a periodic structure,” *Philosophical Magazine* **24**, 145–159 (1887).]

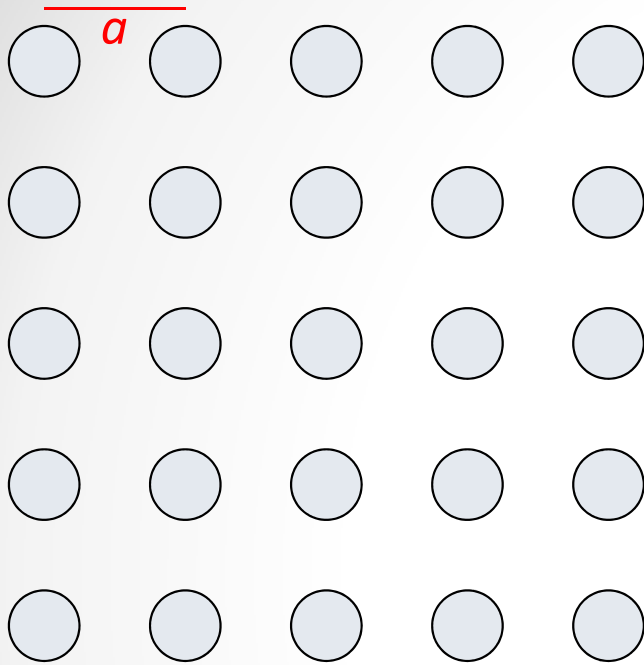
Add a small
“real” periodicity

$$\varepsilon_2 = \varepsilon_1 + \Delta\varepsilon$$

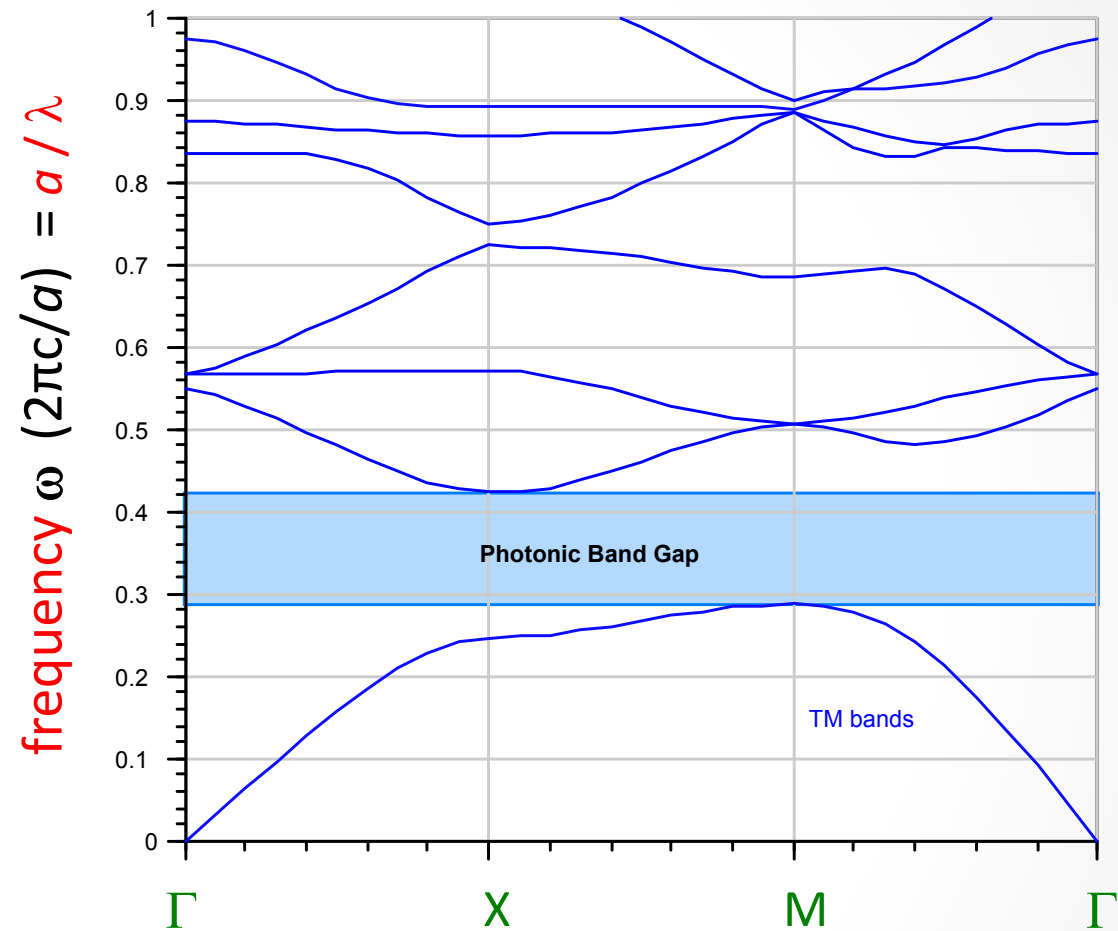
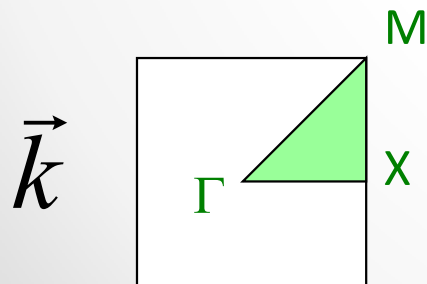
Splitting of degeneracy:

state concentrated in higher index (ε_2)
has lower frequency

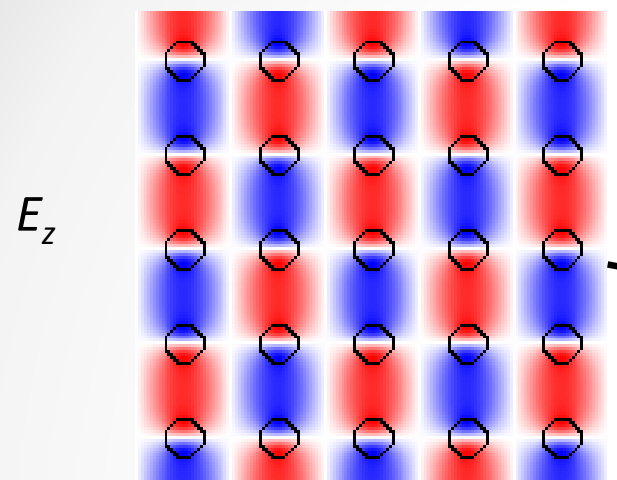




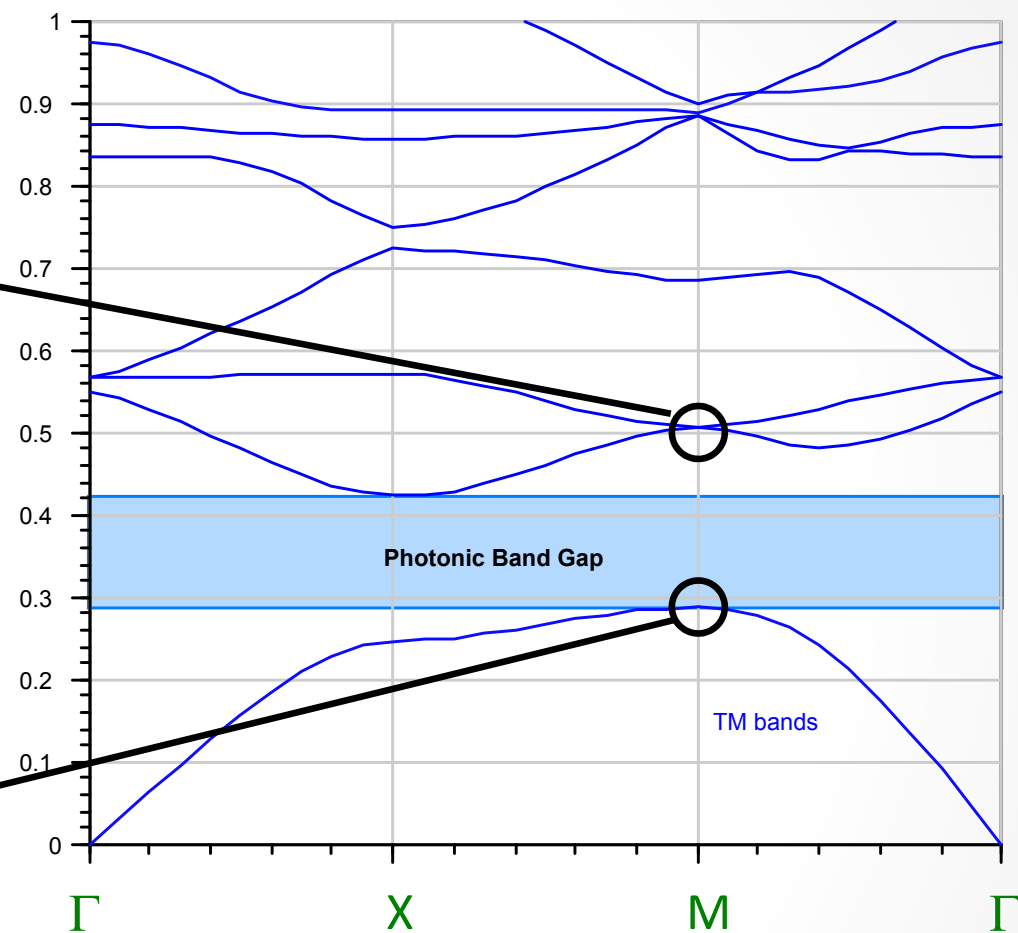
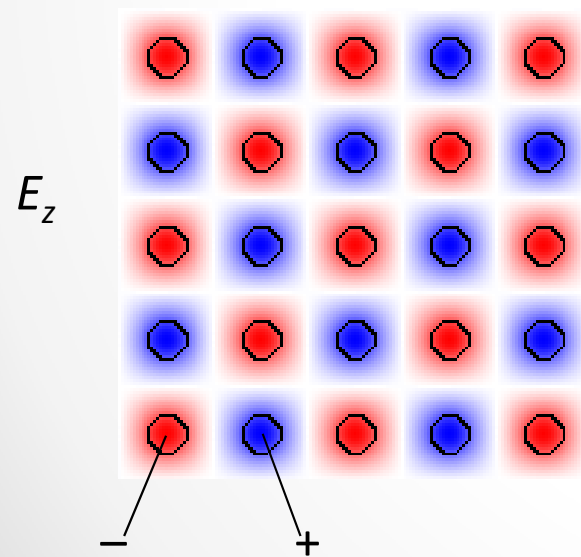
irreducible Brillouin zone



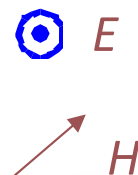
gap for
 $n > \sim 1.75:1$



(+ 90° rotated version)

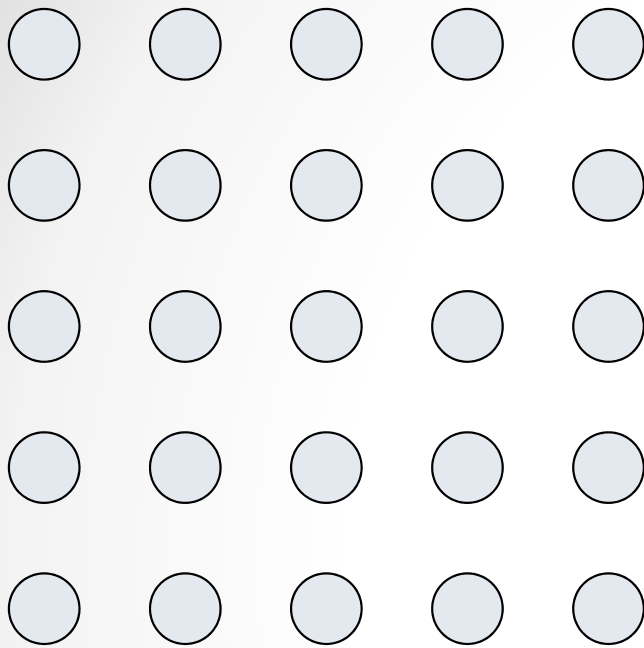


TM

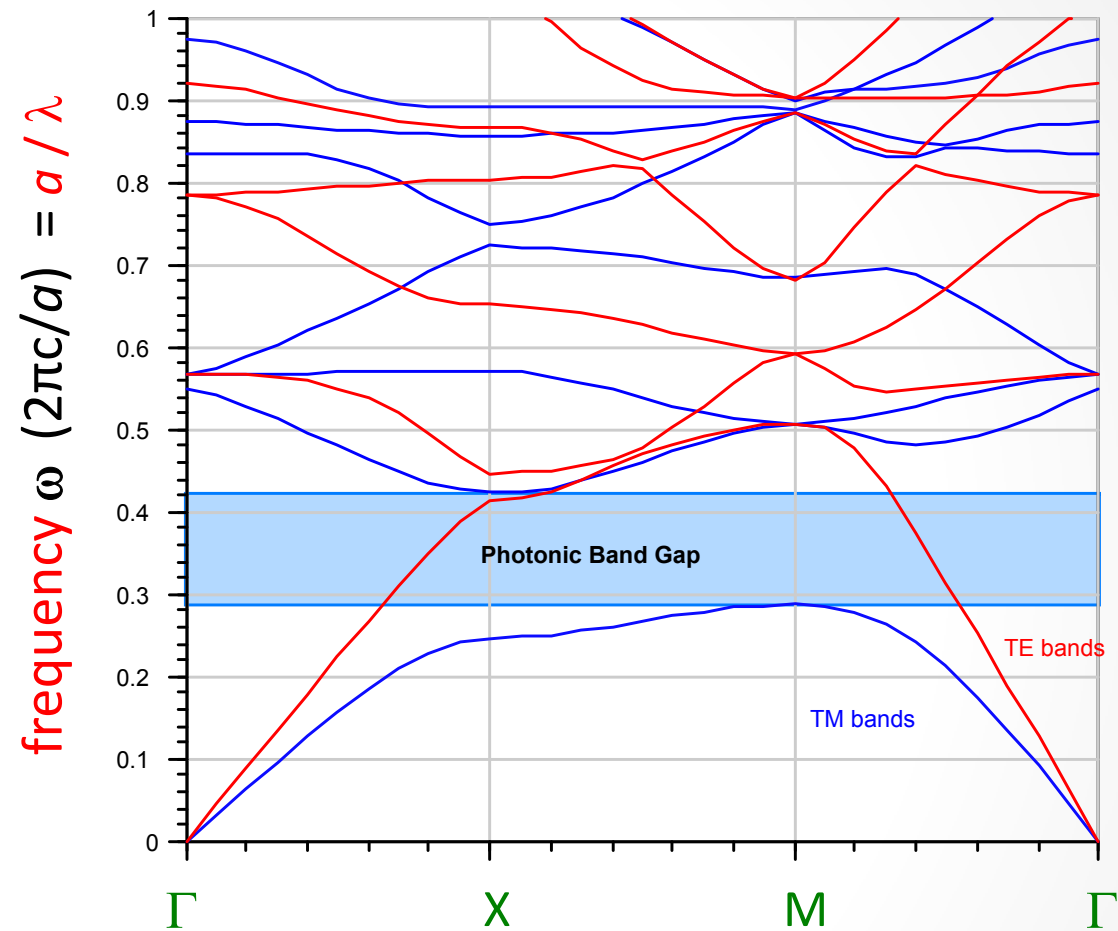
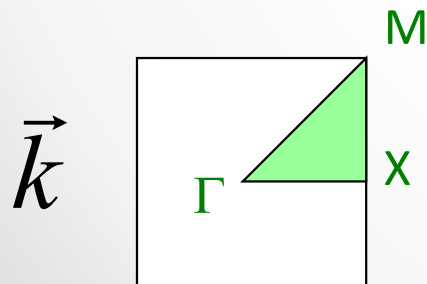


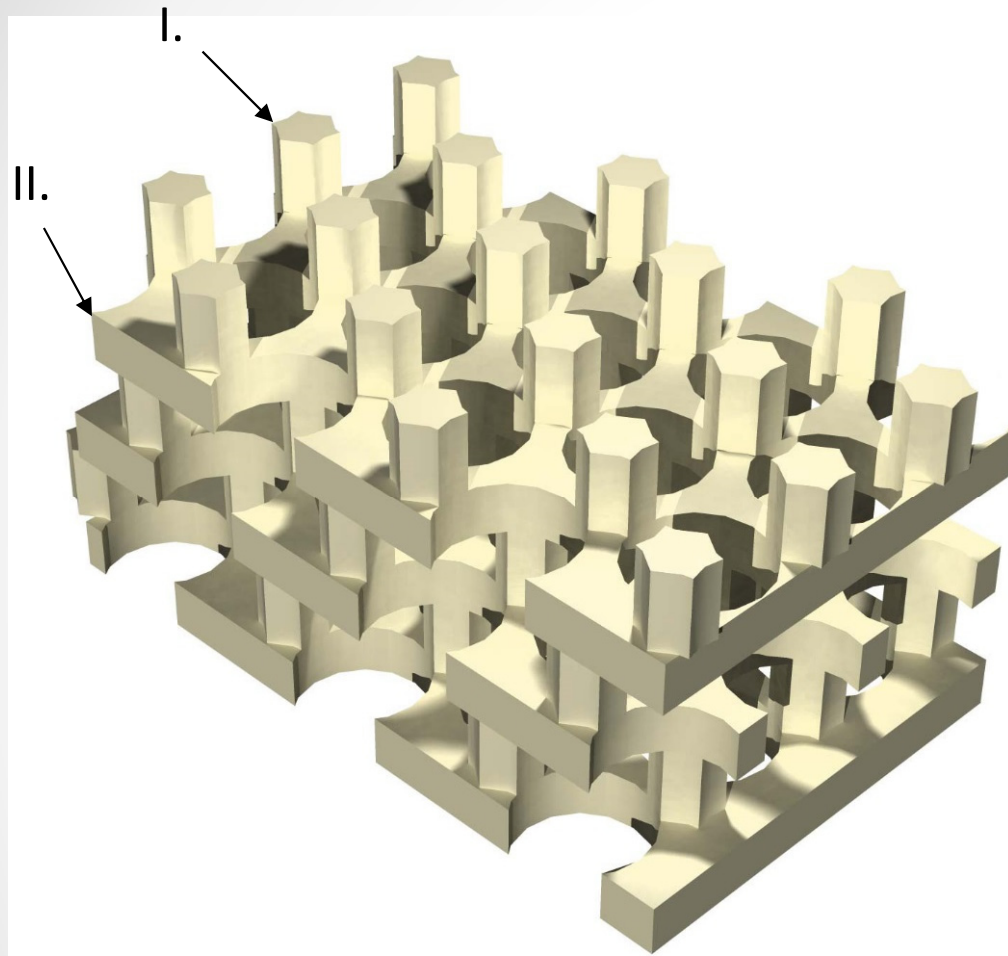
gap for
 $n > \sim 1.75:1$

a

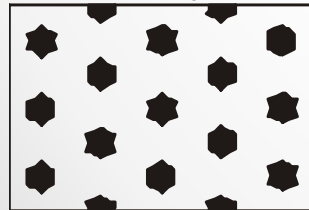


irreducible Brillouin zone

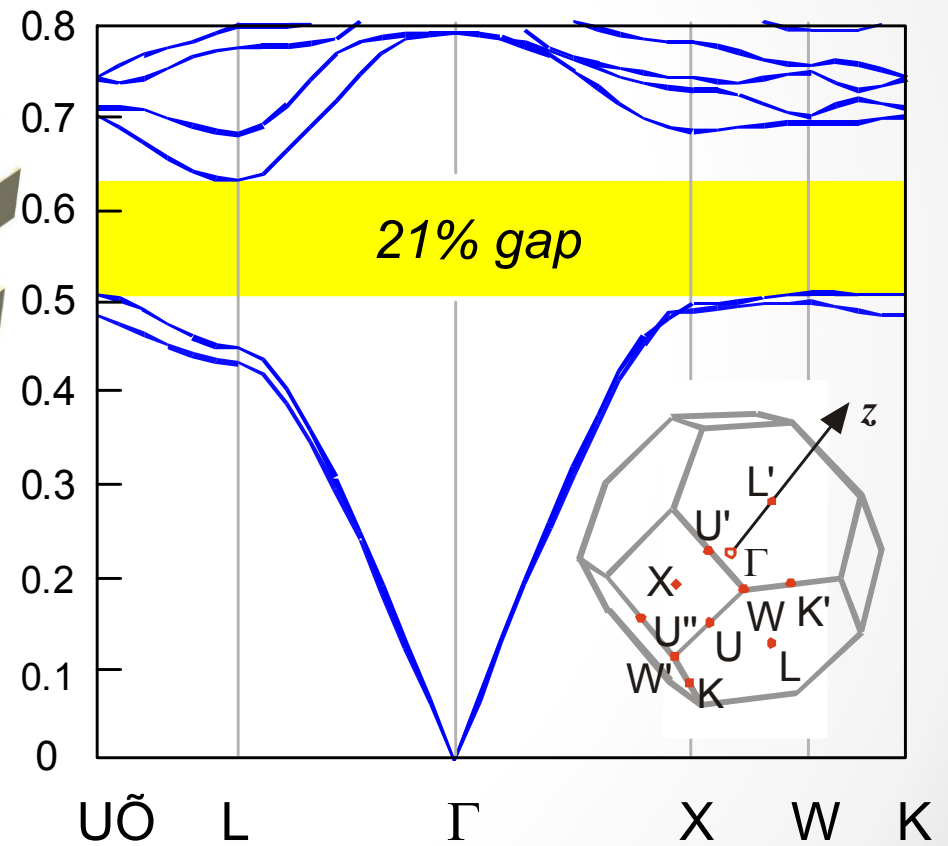


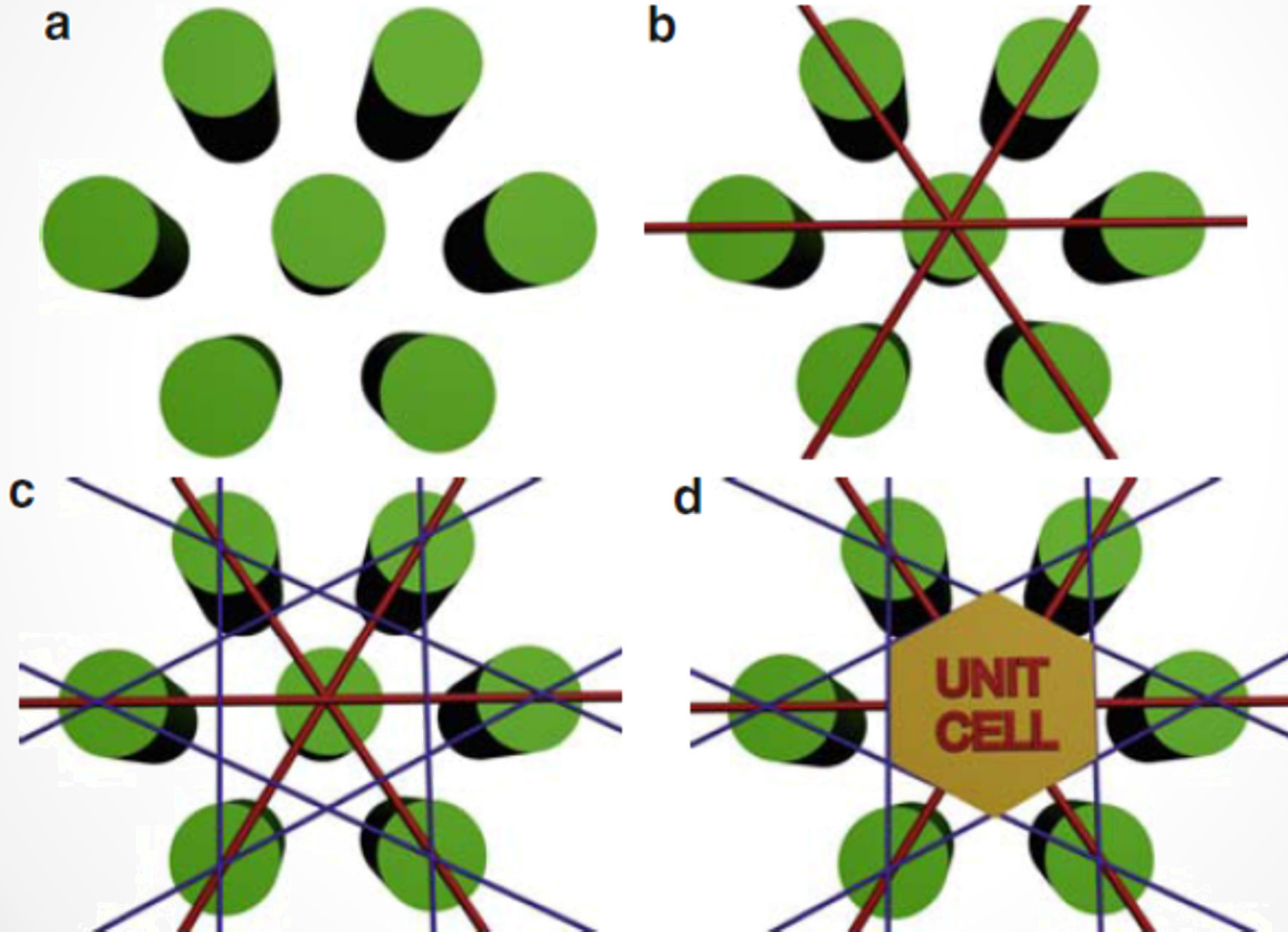


I: rod layer



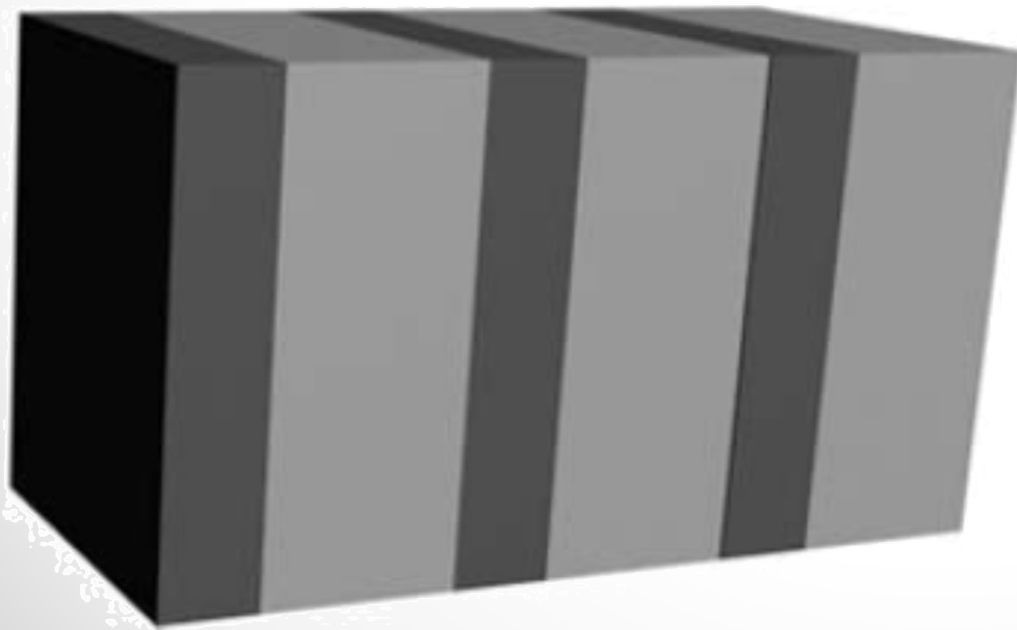
II: hole layer

gap for $n > \sim 4:1$



You, too, can compute
photonic eigenmodes!

1D



2D

