

Diffractive propagation method by a pseudo-spectral approach

Simulation of a Gaussian beam propagating through an optical element

Problem

You have to implement a beam propagation method to solve the equation propagation in the paraxial approximation. It writes when the index variation Δn is not taken into account :

$$\begin{aligned}\frac{\partial \psi}{\partial z} &= \frac{1}{2ik_0} \nabla_t^2 \psi - i\Delta n k_0 \psi = (\hat{D} + \hat{S})\psi \quad [\text{Linear case with } \Delta n] \\ &\sim \frac{1}{2ik_0} (\nabla_x^2 + \nabla_y^2) \psi = \hat{D}\psi \quad [\text{Linear case with } \Delta n \sim 0]\end{aligned}$$

Solution of the propagation equation

The solution of the electric beam modified by the diffraction operator only becomes

$$\begin{aligned}\psi(x, y, z + \Delta z) &= \exp(\hat{D}\Delta z)\psi(x, y, z) \\ &= \mathcal{F}^{-1} \left\{ \exp \left[i \frac{(k_x^2 + k_y^2)\Delta z}{2k_0} \right] \mathcal{F} \{ \psi(x, y, z) \} \right\}\end{aligned} \quad (1)$$

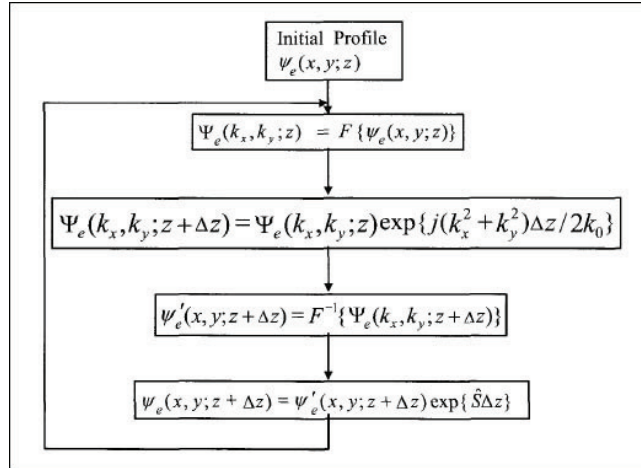


FIGURE 1 – Split-step diagram for the beam propagation method

The method repeats the scheme up to the final distance z is reached.

Focusing by a lens

You will implement the focusing of a Gaussian beam by a thin lens. This focusing corresponds to a modification of the electric phase such that, after the lens, we have $\psi = \psi \times t(x, y)$, where $t(x, y)$ is a function changing the phase only. For an ideal focusing lens, with a focal f , we will consider

$$t(x, y) = \exp \left\{ j \left[\frac{k_0}{2f} (x^2 + y^2) \right] \right\},$$

where $k_0 = 2\pi/\lambda$ represents the wavevector and λ the wavelength.

Geometry and Source Definition

The propagation of the beam along z is modeled using the BPM (Beam Propagation Method) under the paraxial approximation. The initial electric field at plane $z = 0$ is defined as a Gaussian beam :

$$\psi(x, y, z = 0) = E(x, y) = \frac{1}{\omega_0 \sqrt{\pi}} \exp \left(-\frac{x^2 + y^2}{\omega_0^2} \right),$$

where ω_0 is the waist radius of the beam.

In this exercise, the following parameter values will be used :

$$\omega_0 = 50 \text{ } \mu\text{m}, \quad \lambda = 1.03 \text{ } \mu\text{m}, \quad f = 10 \text{ mm}, \quad Z_{\max} = 30 \text{ mm}.$$

Assignment : Propagation of a Gaussian Beam Through a Lens

1. Define the spatial grid (x, y) and the corresponding spatial frequency grids (k_x, k_y) needed for the Fourier-domain implementation of BPM.
2. Generate the initial Gaussian field $Gau_{\text{ini}}(x, y)$ at $z = 0$.
3. Apply the thin lens phase mask to obtain the field $Gau_L(x, y)$ just after the lens.
4. Compute and print the energy of the initial field and the final propagated field to verify energy conservation.
5. Propagate the beam along z using the Fresnel transfer function in the spectral domain.
6. Plot the transverse intensity distribution at the lens plane and at the final propagation distance $Z_{\max}$.
7. Extract and plot the beam width evolution along z (for example by tracking the position where the amplitude drops to $1/e$ of its maximum).

Extension : Propagation through a single silicon cone

In this extension, we study the propagation of the focused beam (from the previous section) through a *single conical structure* made of silicon and placed on a flat silicon surface. The goal is to observe how the beam is transmitted and reshaped after leaving the cone into air.

Geometry

- A flat silicon interface is located at $z = 0$.
- A single cone is centered at $(x = 0, y = 0)$ and extends into air over a height of $h_{\text{cone}} = 10 \mu\text{m}$.
- The cone base diameter is $D_{\text{base}} = 5 \mu\text{m}$ at $z = 0$ and linearly tapers to $D_{\text{top}} = 1 \mu\text{m}$ at $z = h_{\text{cone}}$.
- The cone is at a center of a "pool" of $10 \mu\text{m}$ diameter.
- The refractive index of silicon is n_{Si} (use literature value at the chosen wavelength), and $n_{\text{air}} = 1$.

Propagation scenario

- The focused Gaussian beam reaches the rear silicon surface and propagates forward.
- After crossing the flat silicon interface and the cone volume, the field emerges in air.
- Simulate the transmitted beam in air up to $z = 200 \mu\text{m}$ above the cone tip.

Numerical task

1. Define a refractive index map $n(x, y, z)$ that describes one single silicon cone inside a pool.
2. Inject the focused Gaussian field that fill the pool and the cone and propagate using your BPM code including the $\Delta n(x, y, z)$ phase term.
3. Visualize the beam cross-section in air at different z positions (from 0 to $200 \mu\text{m}$ above the cone).
4. Plot a longitudinal (x, z) cut showing how the beam is modified by the cone.
5. Comment on the beam distortion or focusing/defocusing effect induced by the conical structure.
6. Explain the experimental results, in particular concerning the observed ring.
7. Upload your matlab/python files and a pptx file on ENT showing your results and comments.