

Implementation of the FDTD method

1st example at one dimension

Context :

In this 1st simple example, you have to propagate an EM field assuming a wave traveling along the z direction, with the electric field E polarized along x and H oriented in y -direction. Firstly, the wave will propagate in a free space (vacuum) and then in a material with specified optical properties. In particular, you will consider the case of dispersive materials, presenting a conductivity σ .

Design the mesh :

The propagation space you have to implement is supposed to measure $L = 5\mu\text{m}$ and will be sampled by $N = 1001$ points. This way, you will suppose that each cell of your grid has a dimension $dz = L/N$. The associated time-step will be fixed at $dt = \frac{L}{Nc}$. To have a reasonable number of iterations, N_{iter} will be limited to 2000.

To reduce and simplify the equations, you will introduce $ae = \frac{dt}{(dz\varepsilon_0\varepsilon_r)}$, $am = \frac{dt}{(dz\mu_0\mu_r)}$ and $as = 1 - \frac{dt\sigma}{(2\varepsilon_0\varepsilon_r)} \times \frac{1}{1 + dt\sigma/(2\varepsilon_0\varepsilon_r)}$ and FDTD algorithm becomes :

$$\begin{aligned} Ex_k^{n+1/2} &= as_k Ex_k^{n-1/2} - ae_k (Hy_k^n - Hy_{k-1}^n) \\ Hy_k^{n+1} &= Hy_k^n - am_k (Ex_{k+1}^{n+1/2} - Ex_k^{n+1/2}) \end{aligned}$$

Source definition :

The field you have to propagate is initially produced by a Gaussian source (in time), punctual in space, with a wavelength of $\lambda = 800\text{ nm}$, and a half width (at 1/e) $\Delta t = 15\text{ fs}$, as

$$E = E_0 e^{-\left(\frac{t-t_0}{\Delta t}\right)^2} \sin(\omega t)$$

Boundary conditions :

To start, we will use simple conditions at the boundaries : $\frac{\partial Hy}{\partial z} = 0$ for the left part and $\frac{\partial Ex}{\partial z} = 0$ at the right part of the domain.

- 1.) Implement the field propagation travelling in vacuum, in plotting E_x and H_y .
- 2.) Try different profiles of optical interfaces (sharp and smooth index transitions) of a material medium, on which the wave will propagate and reflect.

Data : $\varepsilon_0 = 8.854 \times 10^{-12}\text{ SI}$ and $\mu_0 = 4\pi \times 10^{-7}\text{ SI}$