

Application of the FDTD method

Nanophotonic 2D project

Context :

For this project, you will propagate a two-dimensional electromagnetic (EM) field with a single pulse moving along the x -direction. The electric field is polarized along y , and the magnetic field is oriented along z .

Propagation geometry :

The computational box should measure tens of μm , sampled with approximately $N_x = 300$ cells in the x -direction and $N_y = 300$ cells in the y -direction. Each grid cell then has a size :

$$dx = L_x/N_x = dy = L_y/N_y.$$

For sufficient wavelength resolution λ , the cell size must satisfy $dx < \lambda/10$. The number of iterations should be $N_{\text{iter}} \simeq 1000$ for a reasonable computational time.

Source definition :

The propagated field originates from a Gaussian laser pulse in time ($\Delta\tau \simeq 10$ fs) and space (width $\Delta y \simeq 5 \mu\text{m}$), with a wavelength $\lambda = 800$ nm ($\omega = 2\pi c/\lambda$) :

$$E = E_0 \exp \left[- \left(\frac{t - t_0}{\Delta\tau} \right)^2 \right] \exp \left[- \left(\frac{y - y_0}{\Delta y} \right)^2 \right] \sin(\omega t)$$

Boundary conditions : You will use the Perfectly Matched Layers (PML) as proposed in the MATLAB example.

Option 1 : Nanoreliefs response in the near-field

1.) Implement the propagation of fields in vacuum, plotting the E , H fields and intensity. Adjust the colormap for optimal visualization. In particular, discuss whether an E_x component appears and why.

2.) Introduce a material at the center of the computational box. Use tungsten with $n = 3$, $\sigma = 5 \times 10^5$ S/m at 800 nm, and a thickness of 100 nm. Apply the Drude model (see below) for more realistic dispersion.

3.) Add various surface morphologies on the tungsten (bump, spike, rod, hole, etc.) and visualize local electric field enhancement. You may propose interacting or periodic systems.

4.) Quantitative analysis : Instead of just visualizing enhancement :

- **Field Enhancement Factor (FEF)** : $FEF = \max(|E_{\text{local}}|)/|E_{\text{incident}}|$. Plot FEF versus geometry parameters (e.g., bump height, rod gap).
- **Absorption Cross-Section** : Compute the absorbed power per unit length from the Poynting vector :

$$P_{\text{abs}} = -\frac{1}{2} \Re \oint (\mathbf{E} \times \mathbf{H}^*) \cdot d\mathbf{S}.$$

Compare absorbed power for different geometries.

Challenge) In FDTD, dispersion is implemented using an auxiliary differential equation for the polarization current, which is then coupled to Maxwell's equations.

6.) Prepare a summary (maximum 10 slides) comparing the geometries and identify which enhances surface absorption most effectively.

Drude model in FDTD :

To model dispersive metals such as tungsten more accurately, include the Drude model :

$$\varepsilon(\omega) = \varepsilon_{\infty} - \frac{\omega_p^2}{\omega^2 + i\gamma\omega},$$

where ω_p is the plasma frequency and γ is the damping rate.

Option 2 : Photonic nanojet induced by a dielectric sphere

- 1.) Implement propagation in vacuum, plotting E , H fields and intensity. Discuss any observed E_x component.
 - 2.) Place a spherical dielectric material at the center of the computational box.
 - 3.) Vary the nanoparticle size and visualize local electric field enhancement.
 - 4.) Quantitative analysis :
 - **Joule Heating** : $U = \frac{1}{2} \sigma \int |E|^2 dt$. Compare heating for different particle sizes or materials.
 - **Photonic Nanojet Definition** : Define the nanojet quantitatively : $\text{FWHM} < \lambda$, intensity $> X \times$ incident field. Measure the nanojet length and width versus sphere size and refractive index contrast.
 - 5.) Explore the effect of placing two interacting spheres in close proximity on the nanojet formation. Quantify how interference between the two nanojets affects FWHM and peak intensity.
- Challenge** : Compute the Fourier spectrum of the electric field along the nanojet axis and analyze how the frequency-dependent field enhancement varies with sphere size and refractive index.
- 6.) Prepare a maximum 10-slide summary of your results and compare them with the literature.