

Plane Wave Expansion Method for 1D photonic crystal

Structure parameter :

In this 1st 1D example, you have to compute band structure of 1D photonic crystal. Unit cell will consist of 2 layers of permittivities ε_1 and ε_2 . The thicknesses of layers are $l_1 = 200$ nm and $l_2 = 800$ nm and the permittivities $\varepsilon_1 = 1$ and $\varepsilon_2 = 9$, respectively. The lattice constant of your unit cell is then $a = l_1 + l_2$.

Implement the PWE method :

You will define the number of plane waves in Fourier series expansion as $numG = 50$ and, due to the requirement to the Fourier series to be zero-symmetric, actual number of plane waves is calculated by formula $N = 2numG + 1$. You will use variables *countk*, *countG* and *countG1* as counters for wave vector inside the Brillouin zone, reciprocal lattices vector G and G' respectively, which are required for definition of matrix differential operator.

Reciprocal lattices vector G and G' will vary from $-2\pi n/a$ to $2\pi n/a$ with steps of $2\pi/a$.

Permittivity definition :

Fourier expansion of the dielectric expansion in 1D case is written down as follows :

$$\frac{1}{\varepsilon(x)} = \sum_G \chi(G) \exp(iGx) = \sum_{l'=-\infty}^{\infty} \chi_{l'} \exp(il'G_0x)$$

Here l' is the number of period of the reciprocal lattice and $G_0 = G/l'$ corresponds to the length of the primitive reciprocal lattice vector. χ_n are Fourier expansion coefficients which are determined in following manner :

$$\chi_n = \chi \cdot (l' \cdot G_0) = \frac{1}{a} \int_{x_1}^{x_2} \frac{1}{\varepsilon(x)} e^{-il'G_0x} dx$$

where a is the period of the PhC in coordinate space and x_1 and x_2 are coordinates of starting and ending points of a single PhC period. In the simplest case, the inverted permittivity profile or dielectric function is step-like, that is, it has the following mathematical description : $\frac{1}{\varepsilon_1}$ for $x = (0+ma; l_1+ma)$ and $\frac{1}{\varepsilon_2}$ for $x = (l_1+ma; a+ma)$ with $m \in \mathbb{Z}$. In case of such a profile it is possible to find the analytical expressions for the Fourier expansion coefficients by substitution of $1/\varepsilon$ and we obtain :

$$\chi_{l'} = \frac{i}{al'G_0} \cdot \left[\frac{1}{\varepsilon_1} (e^{-il'G_0l_1} - 1) + \frac{1}{\varepsilon_2} (e^{-il'G_0a} - e^{-il'G_0l_1}) \right]$$

Loop for band structure calculation :

To compute with all expansion coefficients, you will implement an external loop for scanning over the wave vector inside the Brillouin zone. Boundaries of the Brillouin zone are $-\pi/a$ and π/a . However, due to symmetry a half of the Brillouin zone is enough to be considered.

- 1.) Implement the band structure calculation by solving the eigen-values of the 1D system as a function of the wave vector k to plot dispersion relation.
- 2.) Try different values of ε for the two layers and observe the occurrence of band gap.